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Rayelynn Brandl
Julie Herron
Editors

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The School Science and Mathematics Association [SSMA] is an inclusive professional community of researchers and teachers who promote research, scholarship, and practice that improves school science and mathematics and advances the integration of science and mathematics. SSMA began in 1901, and for more than 115 years, SSMA has provided a venue for many of the most distinguished mathematics, science, and STEM educators to offer their presentations of research at our convention and publish their manuscripts in our journal and proceedings. The proceedings of the 119th Annual Convention serve as a testament to the Association's rich traditions and promising future. In light of the Association's first ever virtual convention due to the COVID-19 pandemic, this rich tradition caused me to reflect upon the ways in which the Association and related research, publications, and conventions addressed and responded to previous historical pandemics.

The 1918 influenza pandemic infected almost one-third of the world's population, and the number of deaths were estimated to be at least 50 million worldwide with almost 700,000 deaths occurring in the United States. With no vaccine to protect against influenza infection and no antibiotics to treat the secondary infections associated with the infection, control efforts were limited to quarantine, isolation, use of disinfectants, wearing of masks, limitations of public gatherings, and good personal hygiene, and these efforts were applied unevenly (CDC, 2018).

These eerily familiar descriptions led me to an exploration of our journal during the time of the 1918 epidemic. Though unable to find any articles specifically addressing the 1918 influenza pandemic, I was able to locate a brief commentary by an unknown author in the October 1912 volume of *School Science and Mathematics*. Titled "Epidemics of So-Called Influenza", the commentary recalls the influenza pandemic of 1889-90 "when within one year the whole civilized world was afflicted with the contagion" (p. 592). Following a brief description of lesser outbreaks classified as influenza epidemics and a word of caution in utilizing the classification without satisfactory confirmation by bacteriologists, the author concludes that, "It is to be hoped that in the future such epidemics in various cities will be more systematically and carefully investigated" (p. 592).

I hope you will join me in applauding the astuteness and foresight of our former member, for these words spoken more than 100 years ago still ring true today and stand in tribute to our Association and its members. Let me also applaud and thank you and all SSMA researchers and teachers for conducting and committing in writing your thoughts, results, and reflections, for our works and words have an impact, and you never know how your words and actions may pique the interest or propel the vision of individuals today, tomorrow, or 100 years from now.

Suzanne M. Nesmith

Suzanne Nesmith
SSMA Past-President

Centers for Disease Control and Prevention. (2018, March 21). *History of 1918 flu pandemic.* https://www.cdc.gov/flu/pandemic-resources/1918-commemoration/1918-pandemic-history.htm

PREFACE

These proceedings are a written record of some of the research and instructional innovations presented at the 119th Annual Meeting of the School Science and Mathematics Association held virtually on November 5-7, 2020. The original host site for the convention was Minneapolis, Minnesota. The blinded, peer reviewed proceedings includes five papers regarding instructional innovations and research. The acceptance rate for the proceedings was 50 %. We are pleased to present these Proceedings as an important resource for the mathematics, science, and STEM education community.

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BROADENING THE PROBLEM SOLVING MEASURES: MOVING ONLINE

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Bostic and colleagues (2015, 2017) explored the validity evidence for a problem-solving measure (PSM) series when administered in a paper-and-pencil format. Any modifications to a measure or the way it is presented that might impact the score interpretations should be examined carefully. The purpose of this manuscript is to explore further development of the PSMs, specifically investigating an online version and fleshing out the validity argument needed to justify their use in online environments.

Keywords: Assessment, middle grades, mathematics

Introduction

Classroom assessments provide opportunities to promote learning and give teachers data about what and how students are learning (Black, Harrison, Lee, & Wiliam, 2004). Since 2009, a majority of states within the United States of America have adopted the Common Core State Standards for Mathematics (Common Core) in some fashion. The Common Core has a clear focus on problem solving (Common Core State Standards Initiative [CCSSI], 2010) and has two equally important components: content and practice standards. The Standards for Mathematics Content (SMCs) describe what students should learn in each grade level. The Standards for Mathematical Practice (SMPs) communicate behaviors and habits students should experience while learning mathematics in classroom contexts. Problem solving is at the core of the SMPs and found throughout every domain in every grade-level SMC. If students are expected to engage in problem solving within the context of the standards, then their problem-solving performance within the context of the Common Core should be assessed using a measure with strong validity evidence. Measurement without strong validity evidence leads to spurious score interpretations (AERA et al., 2014). Searches for such measures usually return empty (Bostic & Sondergeld, 2015). Therefore, there is great need for assessments of this nature to be developed so scholars and school district personnel can use them.

Related Literature

Previously, Bostic and colleagues (2015; 2017) presented the Problem-solving Measures (PSMs), which is a test series that assess middle-school students' problem-solving performance related to the SMCs and SMPs. There are three measures, one each for grades six (e.g., PSM6), seven, and eight, which used Rasch modeling (Rasch, 1960/1980) during test construction. A unique feature of these measures is vertical equating (Bostic et al., 2018). Vertical equating with Rasch modeling is only possible when exploring a single, unidimensional construct (Lissitz & Huyunh, 2003; Wright & Stone, 1979). The PSMs have anchor items that allow test takers' scores from any grade level assessment to be measured alongside a single measurement continuum. Thus, test takers' performance remains on a single scale as students matriculate rather than switching from one test's scale to another. This allows for easy interpretation of scores across years and greater use among schools.

For the PSM series, problem solving has been characterized as a process including "several iterative cycles of expressing, testing and revising mathematical interpretations – and of sorting out, integrating, modifying, revising, or refining clusters of mathematical concepts from various topics within and beyond mathematics" (Lesh & Zawojewski, 2007, p. 782). Problem solving occurs only when learners work on a problem. Schoenfeld (2011) frames a problem as a task such that (a) it is unknown whether a solution exists, (b) the solution pathway is not readily determined, and (c) more than one solution pathway is possible. Problems differ from exercises, which are tasks intended to promote efficiency with a known procedure (Kilpatrick, Swafford, & Findell, 2001). Many have argued that word problems students encounter should be complex, open, and realistic (Bostic et al., 2016; Verschaffel et al., 1999). *Complex* problems require reasoning and persistence because a solution or solution pathway is not clear. *Open* problems allow multiple viable problem-solving strategies and offer several entry points into the task. *Realistic* problems encourage problem solvers to draw on their experiential knowledge and connect mathematics in and out of the classroom. Given these frames for problem solving and problems, coupled with a need for valid, reliable problem-solving assessments, we developed the PSMs to measure students' problem-solving performance within the context of the SMCs and SMPs that allow students' performances to be linked over time.

Objectives of the Study

Validity evidence of these paper-and-pencil measures is available (Bostic & Sondergeld, 2015; Bostic et al., 2017). Score interpretations from the PSMs provide an indication of a student's problem-solving ability as well as a perspective on the degree to which a student understands content described in the Common Core. These are low-stakes tests. Scores are intended to inform teachers' instruction and supplement other data about students' mathematics knowledge and abilities.

Many school districts are moving away from paper-and-pencil tests to online platforms; some have asked about an online version of the PSMs. Online testing is trending because such tests are less expensive, may be scored within minutes, and return with feedback in far less time than paper-and-pencil testing (Paek, 2005). Online administration has potential to change score interpretations; hence, the need for the present study. The research question guiding this study is: To what degree does validity evidence support the use of PSMs being administered online? An objective of this manuscript is to present evidence related to the PSM6, PSM7, and PSM8 when administered using an online platform.

Method

Design

We use the *Standards for Educational and Psychological Testing* (AERA et al., 2014) as a frame for sharing validity evidence. These standards include five sources of evidence: test content, response processes, relationship to other variables, internal consistency, and consequences from testing (AERA et al., 2014). This manuscript reports results from response processes, internal consistency, and consequences from testing. Validity evidence from test content and relationship to other variables are still valid because these areas have not changed.

Participants

Middle school students participated in this Institutional Review Board-approved research. Students' school districts were diverse in nature: rural, suburban, and urban districts. Approximately 40% of the sample came from rural district, 40% came from urban districts, and remaining 20% from suburban locales. In total, 940 sixth-grade, 1006 seventh-grade, and 625 eighth-grade students completed the PSMs in an online environment.

Data Collection and Analysis

Data were collected in two waves. The first wave was a series of cognitive interviews with the intent of gathering response processes and consequences from testing evidence. Teachers recommended students based on ethnicity, gender, and ability, then those students were asked if they wanted to participate voluntarily. The goal was to use representative sampling to achieve a broad understanding about how students might respond to items presented in an online format as well as investigate their perceptions of taking an online test. For each item, students were asked whether they perceived any bias related to the online test. Items were presented by a researcher one-at-a-time to groups of students using an LCD projector. Students were asked to share (a) their perception of any outcomes from online test administration and (b) preference in testing format. In sum, 23 students of those purposefully selected to represent a cross section across the three grade-levels voluntarily participated in cognitive interviews. All names in this manuscript are pseudonyms.

The second wave was PSM administration using an online platform. This second wave followed students' end-of-course testing, hence students were prepared for their grade-level appropriate measure. PSMs were delivered using Moodle, which is an online platform used worldwide as a course medium. Items were presented one-at-a-time and test takers were instructed to type their response to the constructed-response items. This is similar to the paper-and-pencil version where each item is shown on a single page and test takers are instructed to write their final answer to the constructed response items. Students completed the online measures using tablets, Chromebooks, laptops, and desktop machines (both PC and Mac). Similar to the paper-and-pencil format, students took approximately 75 minutes (on average) but were given more time if needed, which may have spanned two class meetings. Students were provided with scratchpaper, pencils, and calculators. They were able to review their responses to any of the 15 items at any time and reminded to check whether they responded to every item.

Qualitative data from interviews were analyzed using inductive analysis (Hatch, 2002). A goal of inductive analysis is to continuously explore data and generate a theme that adequately describes the phenomenon. Quantitative data were analyzed in the same fashion as the paper-and-pencil format. Items were scored dichotomously (correct/incorrect) and analyzed using Rasch analysis (Rasch 1960/1980). Internal consistency was calculated using Cronbach's alpha.

Results and Conclusions

Response processes

A theme from qualitative analysis of interview data was that all students perceived each item to be solvable, readable, and related to content they learned in class. Maria's comment represented a common sentiment across all participants "The questions seem pretty straightforward. I can read them and I have bad eyesight…. I think we did a problem like this [working with expressions and equations] a couple months ago." There were no substantive qualitative differences across participants in their responses.

Internal Structure and Reliability

Quantitative results and percentages (see table 1) indicated that students performed satisfactorily using the online platform in two instances: $M_6 = 3.95$ ($SD_6=3.43$); $M_7 = 6.93$ ($SD_7 = 4.51$); $M_8 = 5.96$ ($SD_8 = 3.81$). The low scores align with the premise shared in previous published work (see Bostic & Sondergeld, 2015, Bostic et al., 2017): problem solving is more difficult than completing exercises. Hence, it is anticipated that students' problem-solving performance might be lower than end-of-course tests that include exercises.

Reliability of the PSMs continued to meet acceptable standards. Cronbach alphas above 0.80 are considered good (Nunnaly, 1978). Cronbach alphas for the online versions of the PSM6, PSM7 and PSM8 were 0.845, 0.880, and 0.826, respectively. This leads to the conclusion that internal consistency of the problem-solving measures in the online platform had appropriate reliability, like the paper-and-pencil versions.

Table 1

Comparison of descriptive statistics for paper-and-pencil and online PSMs

	Mean (SD)		**Percentage (%)**	
	Paper-and-pencil	**Online**	**Paper-and-pencil**	**Online**
PSM6	5.7 (3.1)	3.95 (3.43)	38	26
PSM7	4.88 (3.2)	6.93 (4.51)	26	36
PSM8	3.93 (2.73)	5.96 (3.81)	21	31

Consequences from testing

A qualitative theme from interview data was that test takers preferred to use the paper-and-pencil format; however, they perceived no difference between the paper-and-pencil format and online format. Lance shared "I'm saying, like, the longer a test is, the more I'd go for computer….and you can always go back and change it [your answer]. Short tests are OK unless there's lots of writing, like in English." Tim shared "It [the online test] doesn't overload [the user]…When you get a big packet of a test [like the paper-and-pencil version]…it's overwhelming. Like your head explodes. One problem at a time, it loads one screen at a time. That's OK." Students generally agreed that they were comfortable doing their work on pencil and paper then transferring it. Tim added, "I can just type in my answer after working it out on paper." Given that they could do their work on paper and pencil; typing their final answer was perceived as a trivial step.

Significance of work to field of Research Evaluation and Assessment in Schools

Drawing together the quantitative and qualitative results, the validity evidence for these sources is strong for using the online version of the problem-solving measures. The PSMs administered online appear to have strong evidence in all three examined validity sources. Score interpretations from PSMs administered online may be treated as similar to those score interpretations from paper-and-pencil PSM administrations (see Bostic & Sondergeld, 2015, 2018; Bostic et al., 2017). Districts and researchers may feel confident using the PSMs in an online platform. Such validity studies are needed to inform potential users and administrators about the appropriateness of validated assessment systems. As more districts trend towards online test administration, it is appropriate to investigate and compare online and paper-and-pencil test administration.

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EXAMINING CONTENT AND PEDAGOGICAL KNOWLEDGE OF PRE-SERVICE ELEMENTARY GRADES MATHEMATICS TEACHERS

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This study examines the content and pedagogical knowledge of pre-service elementary grades teachers (PSTs). Our working hypothesis is that a teacher's mathematical content knowledge influences their mathematical beliefs that in turn affects the decisions they make in their instructional activities. Our focus is to examine how the PSTs' conceptual understandings of fraction and part-whole relationships may inform their pedagogical decisions in problem situations that involve proportional reasoning. Specifically, the paper focuses on 1) examining possible connections that PSTs make between their mathematical content and pedagogical knowledge; and 2) documenting these connections in illustrative examples.

The mathematics education community has long emphasized that mathematics teachers need to have strong backgrounds in both mathematics as well as teaching pedagogy (NCTM, 2000). In this paper we look to identify connections between teachers' mathematics content and pedagogical knowledge. Our hypothesis is that teachers' pedagogical approaches are influenced by the ways they view and understand mathematics. While such connections have been hypothesized in the literature, there are challenges in isolating and studying these kinds of connections (Schoenfeld, 2010).

Objectives of the Study

The goal of this study is to examine connections between the content and pedagogical knowledge of pre-service elementary grades teachers (PSTs) in the context of solving proportional reasoning problems. Specifically, the study looked to examine how the pre-service teachers' conceptual understandings of fraction and part-whole relationships inform their pedagogical decisions in mathematics learning situations.

In considering mathematical content, we chose proportional reasoning problems as the content area because 1) They encompass an important area of study that begins in the early grades with the study of fractions and partwhole relationships and extends to formal proportional relationships in the middle grades (Lamon, 2007); and 2) They offered a set of mathematical situations that we expected would be challenging enough for the PSTs so that we could observe some of their problem-solving actions.

Related Research

Content knowledge: Proportional reasoning

While typically studied as reasoning that develops in grades 6-8, children's difficulty with proportional reasoning in the context of conventional fractions in grades 3 and 4 is noted in the mathematics education literature (Ball, 1993; Pitkethly & Hunting, 1996). For example, Ball (1993) reported that third-grade children systematically misinterpret traditionally notated fractions (e.g., ¾), and estimate that fractions with larger denominators are quantitatively greater than fractions with smaller denominators (e.g., 4/8 > 4/6).

Since proportional reasoning has its sources in the early grades, a goal of this study was to assess the proportional reasoning knowledge of pre-service elementary grades mathematics teachers to determine how well they would be able to provide their students with the necessary mathematics foundation for solving proportional reasoning tasks.

Pedagogical knowledge: Assessing student understanding

The connection between a teacher's content and pedagogical knowledge is difficult to isolate and study (Schoenfeld, 1994). According to Schoenfeld (1994), this difficulty is because the act of doing mathematics differs substantially from the act of teaching mathematics to a group of students. Hence, there is no direct link between content expertise and teaching practice. However, having teachers examine student work to assess understanding of students' mathematical thinking has been encouraged as a way for examining how teachers demonstrate content expertise in the context of instructional settings (Brown & Clark, 2006). Specifically, this approach allows teachers to identify and analyze the fundamental mathematics content and processes used by students thus providing them with a basis for making evidence-based conjectures about students' mathematical understandings. Hence, the current study made use of tasks that required the teachers to consider hypothetical problem solutions of students solving proportional reasoning tasks.

Methodology

Participants

The participants in this study (*N*=4) were pre-service elementary mathematics teachers enrolled in a sophomore level mathematics course taught by the first researcher. The participants comprised a diverse subset of the class in terms of ethnic characteristics. The four participants consisted of one white male (Matt), one Latino male (Mario), one white female (Katelyn), and one African American female (Katherine). In addition, the students demonstrated a an unusually strong level of mathematics preparation. These PSTs had taken more mathematics than typical elementary grades teachers. Specifically, the PSTs took on average nine more semester hours of mathematics than is required for elementary grades majors. With this sample we looked to build on the research on PSTs having advanced mathematics preparation (Phillip, Flores, Sowder, and Schapelle, 1994).

Measures and instrumentation

The PSTs each participated in 4 interviews: In Interview 1, we asked of the PSTs a set of questions designed to document their formal background in mathematics as well to probe some of their beliefs and attitudes about mathematics teaching and learning (Figure 1).

Figure 1

Introduction Questions

1. Tell me about your mathematics background in K-12 and college.
2. What are the attributes of a successful mathematics teacher?
3. What are the attributes of a successful mathematics student?
4. If you could change one thing about current mathematics classrooms and teaching, what would it be?

In interviews 2-4, the PSTs solved a variety of content and pedagogical problems (Figure 2). With these tasks, we hoped to observe some important connections between their content and pedagogical knowledge.

Figure 2

Sample Content and Pedagogical Tasks Used in the Study

Content problems involving fractions and proportions	
Task 1	Order the fractions 4/7, 7/13, and 14/25 on a number line.
Task 2	Find a fraction between 5/6 and 11/12
Pedagogical problems based on hypothetical student work	
Task 3	*The Magic Algorithm* • Find a fraction between 5/6 and 11/12 • Suppose one of your students, Christine gave 16/18 as an answer. • Is Christine's answer correct? • How do you think Christine got that answer? • How would you discuss her solution with the class?
Task 4:	*Classroom Ratio Task* (adapted from LMT, 2005) In Mrs. Calabrese' class the ratio of boys to girls is 4 to 5. If there are 12 boys in the class, how many students total are in the class. Erin and Sean responded: **Erin**: In a class with 4 boys and 5 girls, the fraction of boys is 4/9, so I can solve the proportion 4/9 = 12/x **Sean**: The way to represent a ratio like 4 to 5 is by using the fraction 4/5, so I started with 4/5 = 12/x. Please comment on each student's method.

The content problems (Tasks 1 and 2) provided a measure of their knowledge of operations with fractions while the pedagogical tasks (Tasks 3 and 4) provided examples of hypothetical student work on fraction and proportion reasoning tasks and asked the PSTs to assess the students' work.

Data collection procedures

The class instructor conducted the interviews of the PSTs completing the tasks. The PSTs were encouraged to describe their formal experiences as mathematics students and verbally self-report their solution strategies as they completed the content and pedagogical tasks. All interviews were videotaped. Data consisted of the pre-service teachers' verbal and written work and the interviewer's field notes. Written transcripts were generated from the PSTs' video protocols.

Data analysis procedures

To analyze data, we summarized the PSTs' formal mathematical experiences and then examined their work in Tasks 1-4 to identify and classify the various strategies they used to complete the tasks. We looked for common themes across the written work and classified the strategies accordingly. So, for example, if we could observe them demonstrating flexibility and efficacy in their problem-solving actions, then we would expect to find them holding beliefs that value an inquiry-based approach in their teaching views.

Results and Discussion

The results are reported as follows. First, we provide an overview of the PSTs' mathematical experiences and beliefs. Second, we briefly describe the PSTs' work on the content tasks (Tasks 1 and 2). Third, we summarize and illustrate the PSTs' work on the pedagogical tasks (Tasks 3 and 4).

Mathematical experiences and beliefs

The PSTs' mathematical experiences and beliefs taken from the introduction questions are summarized in Table 1.

Table 1

Overview of the Pre-service Teachers' Mathematical Experiences and Beliefs

Name	Highest math completed	Class Grade	Question 2	Question 3	Question 4
Matt	Calculus	A	Understand pure numbers and number operations	Knows the material Open to alternative PS approaches	De-emphasize the textbook and have students provide more explanations
Mario	Calculus	B	Can think outside the box to solve problems in different ways	Must maintain open mind to students' actions Must look to students' interests	Implement more activities in the classroom. Students will apply concepts that interest them
Katelyn	Pre-Calculus	C+	Uses all available as needed	Must be able to relate to all of the students	Provide more resources for teachers
Katherine	Calculus	A	Can think critically to solve and explain problems;	Must be able to relate different approaches Must be approachable to all students	Pursue a more holistic approach that focuses on making connections

The PSTs' responses comprise an interesting range of views about mathematics teaching and learning. For example, Matt and Katherine consistently remarked on the importance of the teacher having a strong grasp of the mathematics in order to help students see connections among various topics and the importance of helping their students to think critically to solve problems.

Matt: As a teacher, my goal is not to teach them math but to teach them how to solve problems. If ideally my students could do their HW and teach themselves, then I could bring them back together and discuss what they know about it.

Katherine: It is important for teachers to have a holistic approach for students to learn connections between functions and equations. The textbooks never connect the pieces.

In contrast, Mario and Katelyn emphasized the importance of the teacher providing for students a strong foundation upon which to build increasingly abstract concepts. Though a bit more focused than Matt and Katherine on teaching 'the mathematics' that students need, they each held some very interesting views about what their students would need to be successful.

Mario: I think it is important for students to learn basics and then explore many different problems. If they do not explore, they will not learn. Exploration is the key!

Katelyn: The basic operations are so important. That is not all you learn but they are things I learned in grades 1 and 2 that I am still using.

Mathematical content knowledge

Three of the four pre-service teachers used equivalent fractions to solve the content tasks. Of these, only Matt used prime factorization to find the least common multiple of the denominators of the equivalent fractions, commenting, "they will need this approach (finding the Least Common Denominator) later on." Only Katelyn was unable to use equivalent fractions to complete the tasks. She was the only participant who used a calculator to compute answers.

Pedagogical knowledge

The PSTs solutions to Task 3 (Magic Algorithm Task) and Task 4 (Classroom Ratio Task) are summarized in Table 2.

Table 2

Pre-Service Teachers' Performance on Pedagogical Tasks

Name	Task 3	Task 4
Matt	Acknowledge the student's contribution as a possible new method (and would "do some homework before next class to find out why it works!")	Correct solution, that both students' approaches could be used; Viewed student solutions as equally appropriate to solve the problem
Mario	Acknowledge the student's contribution as awesome! Thinking differently can pay off	Eventually acknowledged that both students could be correct
Katelyn	Acknowledge the student's contribution as a shortcut for solving the problem	Erin was incorrect; Upon reflection, she maintains she is correct
Katherine	Acknowledge the student's solution as interesting but quickly move on – it is haphazard and not mathematical!	Correct solution, that both students' approaches could be used; Prefers Erin's approach because it yields the solution directly

Given the PSTs' strong mathematics background, we found the PSTs' performance on the Magic Algorithm task to be somewhat surprising: we expected all of the PSTs, based on their strong mathematics background would be critical of the student's solution because it appeared to be more of a trick rather than based on sound mathematics. However, only one of the participants, Katherine, thought the student solution to be problematic in terms of conceptual understanding.

Katherine: How about 1/3 and 2/3, so we get 3/6 = ½, yes it works. Let's try 2/4 and 3/4, we get 5/8 and yes, it works. I would be hesitant for her to get into the habit of it because it would make me nervous – that is not how we add fractions! I would discourage her from doing it. Just because it works doesn't mean it will always work. There is no understanding, seems haphazard! It solves this general problem but will not solve the harder problem of finding one halfway between.

Mario: I think Christine is awesome! I would applaud her with the other students.

Katelyn: Christine has found a shortcut; yes, it is good that students can see different ways.

The Classroom Ratio task was much more difficult for the PSTs since it required them to assess the solutions of the two students, Erin and Sean. Matt started by solving the problem and developed the most complete solution of all of the participants. Based on his solution, he was not at all concerned with the difference between the two student solutions:

Matt: Let me solve it. Therefore, we have 4 to 5 boys to girls. Therefore, if there are 12 boys we get 15 girls. It asks for total, total is 27. You would have 15 girls and you add the two. I think they both work. He needs to add his up and get the total number. Sean's is equivalent to mine. Erin did boys to total students so she got it right. So, Sean just needs to add them up in the last step.

In contrast, the other students did not first solve the problem; rather they commented directly on the students' solutions and thus struggled more than Matt in solving the problem. For example, Mario was only able to solve the task after much effort:

Mario: I think that neither Erin nor Sean is correct. Erin has the wrong ratio. I would just do this in my head. I would know if it is 4 to 5 For every 4 there are 5 if you have 12 then 4/12=5/x, what, that doesn't work either. I guess 4/12 =5/x 4x=60 divide by 4, is it 15. I guess that does work. If it is 4 to 5 students …. No, Sean would be right. Sean is right but it doesn't gibe with the number of students. I see what Erin is doing. So, yes, Erin is right also.

Katherine acknowledged that both students could be correct but that she preferred Erin's approach because it yielded the solution directly. Finally, Katelyn never ventured from her initial intuition that Erin was incorrect.

Katelyn: What she wrote, the ratio is 4 boys and 5 girls and she set it up, there are 12 boys, she did not put the boys with the boys. I thought it is 4/5=12/x, so Sean is right. I think she is wrong because 4/9 …. the fraction of boys, I do not know where she got 9! Did she add 4 and 5? Did she think it was the total? That is what I think. She is confusing fractions and ratios.

Implications

We must be careful not to conclude too much from these findings since ours was a limited sample. In addition, while the findings hint at some important connections between the PSTs' content and pedagogical knowledge, we cannot conclude with certainty that these are robust. More work is needed in this area.

We believe that some observations are noteworthy. First, the PSTs' ideas about teaching and learning were very rich and certainly confirm the importance of elementary grades teachers having a strong background in the mathematics they will teach (Schoenfeld, 1994; 2010). Second, we are reminded that even with advanced mathematics preparation, PSTs can be successful in many different ways. While we observed two of the PSTs, Katherine and Matt appeared to be more focused on the importance of students developing as autonomous problem solvers, Mario and Katelyn's emphasis on teaching 'the mathematics' by no means suggests that they would become rigid teachers. Both PSTs stated the importance of providing challenging tasks to their students and staying vigilant for diverse problem-solving approaches that students might demonstrate.

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INTERTWINING MEASUREMENT, DATA AND GEOMETRY CAEP K-6 STANDARDS INTO A MATHEMATICS COURSE

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Planning lessons to intertwine mathematical content provides deep learning opportunities for preservice teachers. From the CAEP K-6 elementary teacher preparation standards, a contextual argument is made to intertwine data concepts, specifically probability, into measurement and geometric activities. This paper presents a handful of pedagogical activities to utilize with a classroom of elementary preservice teachers.

Keywords: probability, teacher education, innovative practices, statistical technology

Introduction & Purpose

The ability to use quantitative reasoning often separates an individual's ability to successfully problem-solve challenges encountered throughout life. Therefore, universities typically require their undergraduates to complete a quantitative reasoning course. As society changes and the global economy develops, quantitative thinking in a statistical context is emerging as an essential workforce skill.

The purpose of this paper is to describe a handful of pedagogical activities that intertwine content typically considered to be categorized as data and statistics content into the measurement and geometry standard domains. Individuals who teach data and statistics courses typically work in mathematics departments at either the university or high school levels, with some exceptions in psychology and the sciences. However, there are substantial differences between the core discipline of mathematics and the methodological discipline of statistics (Cobb & Moore, 1997), leading statistics educators to call for teaching statistics differently than other STEM courses (American Statistical Assoication, 2015). Unfortunately, universities struggle to adjust coursework to focus on (or in some cases even include) classes in statistics. One reason for limited statistics coursework is the credit-size requirements of majors or minors in education already require four-years of full-time college coursework to complete. Secondary math preservice teachers are required to cover coursework in many different fields of mathematics, while elementary teachers are generalists and are required to learn a spectrum of subjects across many grades (i.e. Language Arts, Science, Math, etc.). The opportunity to learn statistics is at a critical juxtaposition; employers desire to hire professionals that are explicitly trained to use statistics while problem solving in their respective field, but adjusting collegiate coursework to provide sufficient statistical training requires a herculean effort, often through curricular change. Therefore, many undergraduate programs, including teacher education programs, continue to embed statistical content in current coursework as a modus vivendi.

Instructional Framework

Teaching mathematical content with pedagogical tactics that facilitate learning for all students is critical to conducting a successful classroom. Therefore, national organizations (American Statistical Association, 2015; Conference Board of the Mathematical Sciences, 2012; Common Core State Standards Initiative, 2010) produce documents that provide direction through standards and instructional commentaries that is repurposed by accrediting bodies to help teacher education programs. One accrediting body, the Council for Accreditation of Educator Preparation (CAEP) published K-6 Elementary Teacher Preparation standards (2018) that many teacher education programs utilize to continually improve practices. An important pedagogical articulation found in these standards vis-à-vis the aforementioned national organizations and documents is the importance of connecting mathematical content domains while enacting the eight mathematical practices. CAEP Component C.2.3, standard 2.b states: "Candidates demonstrate and apply understandings of major mathematics concepts, algorithms, procedures, applications and mathematical practices in varied contexts, and connections within and among mathematical domains." (2018, p. 10). Standard 2.b continues to list off the major domains of mathematics (albeit with limited attention to probability) followed by the Mathematical Practices that describe how to teach content. The broad and open-ended phrase of *connections within and among mathematical domains* places emphasis on the need to purposefully design classroom activities to intertwine content domains. Fortunately, constructivist learning environments by design accomplish this emphasis in their very essence. A constructivist learning environment involves a deep interaction between participants and content, and for preservice teachers further intertwines pedagogical aspects. Some educators described the learning process preservice teachers encounter as a "spiraling," or a switching between methods, pedagogical techniques, and depth of content understanding (Martin & Jones, 2019). For the data and measurement content domains, there is often an interconnection or "spiriling" that educators easily notice. Perhaps less obvious, but still feasible to interrelate are the data and geometry domains.

Classroom Examples

The following section provides a handful of classroom examples and pedagogical techniques that demonstrate connections within and among mathematical domains. Both of the described activities provide a teacher with opportunities to intertwine data concepts, specifically probability, into measurement and geometric activities. Embedding probabilistic ideas into the established measurement and geometric domains provides preservice teachers with much needed exposure to probabilistic content.

Angles and Spinners Activity

Articulating definitions of relatively simple geometric ideas is often challenging for preservice teachers. Ask your preservice teachers to define what an "angle" is, and responses often return as "something measured in degrees." Preservice teachers can read the article *What's your angle on angles?* which argues an angle has three potential meanings: 1) the static notion of two rays meeting at a common vertex, 2) as the space between two rays or a wedge, and 3) a dynamic idea of a turn (Browning et al., 2007). With preservice teachers now prepared to think about angles in multiple, flexible manners, an opportunity arises to rethink how critical the angle measurement is in a spinner. (Daiga & Kloosterman, 2019). Spinners often come in standard shapes that unfortunately do not highlight that *central angles* are the key aspect to calculating probabilities on a spinner. Provide students with an unstandardized spinner (see *Figure 1*) and ask preservice teachers, "Is this spinner fair? Would young children think this spinner is fair?" The spinner shown includes four colors all of which share a common angle measurement of 90º or one-fourth of the entire spinner. The areas of each of these sections differ substantially with the orange section covering far-more area than the rest of the colors. Highlight the visual inequities between spinner angle measurements and section areas by having preservice teachers calculate each area on the spinner.

Figure 1

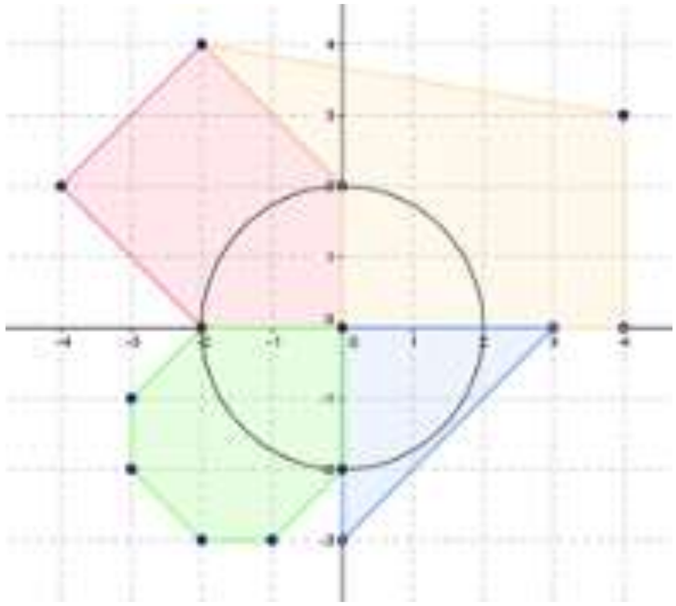

Figure 1: Rethinking Spinners example (Daiga & Kloosterman, 2019, p. 271)

If time permits, you can extend this activity by asking preservice teachers to create three types of spinners: 1) one with equal angles and equal areas, 2) one with equal angles but unequal areas, and 3) one with unequal angles and unequal areas. (Daiga & Kloosterman, 2019, p. 272) After displaying spinner creations be sure to emphasize that angle measurements, not areas, dictate the chance of landing on spinner sections. When preservice teachers are challenged to work with unstandardized spinners, they articulate the connection between three different mathematical topics: angles, areas, and probability.

Fair Roll Activity

Teachers and researchers utilize dice in different manners to help students think about probability and chance concepts (Truran, 1995; Flores, 2006). With the help of a 3D printer, design and create a set of dice, or now cuboids, that are stretched in a direction. The dimensions of 2x2x2cm, 2x2x2.5cm and 2x2x3cm work well. Ask preservice teachers "How do these cuboid dice roll and land?" After allowing a few moments for preservice teachers to discuss this question and think about possible solutions, provide some structure for preservice teachers to organize their thoughts through a prediction handout (see *Figure 2*). At the end of the activity, ask preservice teachers to write a short persuasive paper to justify their stance on how the cuboids will roll and land. Although clear expectations for the persuaive paper with regards to length and written communication should be provided, other criteria focused on pedagogical content knowledge should be assessed as well. For example, asking preservice teachers to provide specific descriptions of the evolution of their thinking or requiring preservice teachers to describe their observation of a groupmates probabilistic thinking while completing the task can help evaluate preservice teachers' pedagogical thinking.

Figure 2

Fair Roll???

As a group, you have been given 3 uniformly distributed plastic shapes, a 2x2x2cm cube, 2x2x2.5cm rectangular prism, and 2x2x3cm rectangular prism. Your task is to build a persuasive argument describing how often each face (1,2,3,4,5 or 6) will land facing up. **There are many different ideas/answers you can use to create your argument, however you need to justify your arguments logically.**

Let's Start by **PREDICTING** the chance each shape will land on the corresponding face in the table below:

	Face 1	Face 2	Face 3	Face 4	Face 5	Face 6
2x2x2 cube	____%	____%	____%	____%	____%	____%
2x2x2.5 rectangular prism	____%	____%	____%	____%	____%	____%
2x2x3 rectangular prism	____%	____%	____%	____%	____%	____%

Figure 2: A handout to help preservice teachers predict how cuboids will roll and land.

While solving the Fair Roll activity. preservice teachers often use different meanings of probability (Batanero & Diaz, 2009; Albert, 2006) including classical (i.e. theoretical), frequentist (i.e. empirical), and subjective (i.e. personal or intuitive) meanings. Preservice teachers develop arguments of how cuboids roll and land based on different lenses of probability, guided by their beliefs and not necessarily mathematical or statistical content. For example, many preservice teachers will intuitively believe the stretched cuboids will not roll fairly but will struggle to articulate a meaningful reason to explain their stance. Eventually, most preservice teachers begin to use the Law of Large Numbers and predict cuboids landing on certain numbers based on each face's surface area in proportion to entire cuboids surface area. Prediction in this manner helps align a conversation between different meanings of probability and critical geometric vocabulary to utilize with elementary students (i.e. faces, edges, vertices, rectangular prisms).

Figure 3

Figure 3: Example cuboids made with a 3D printer and then painted.

A possible extension of this activity could involve creating cylindrical objects to roll from dowel rods (Jones, 2009). Cylindrical objects can highlight what π actually is (i.e. the relationship of circumference of a circle being divided by the diameter) and if edges on a 3D object by definition can be curved. Preservice teachers could even design and create a unique, rollable object of their choice. As the complexity of the rollable object increases so does the complexity of the modeling process. For example, predicting how a *hollowed-out* cuboid rolls and lands changes the object's weight distribution causing them to land hollow-side up more often (see *Figure 4*). Preservice teachers who modeled based on surface area calculations must reconsider their strategy.

Figure 4

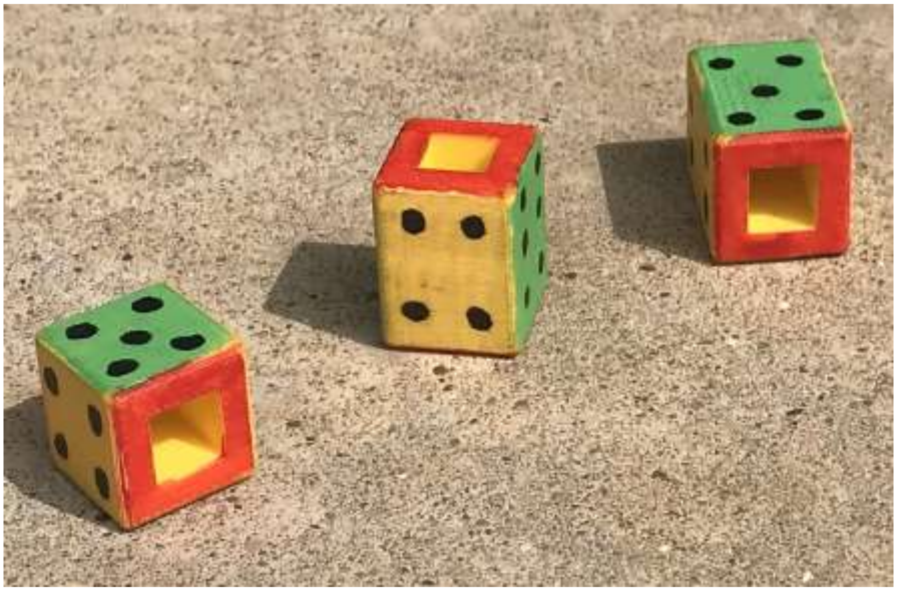

Figure 4: Hollowed-out cuboids.

With technological programs becoming more critical to the classroom each year, a pedagogical technique to reinforce the topics discussed with the Fair Roll activity is utilizing a dynamic statistical software. The Common Online Data Analysis Program (CODAP, 2018) is a free online resource designed for grades 6-14, to improve data literacy in the classroom. CODAP allows the user to interact with the data in a variety of manners, many of which would take hours (if not days) to complete by hand. A prebuilt CODAP task called *Investigating the Fairness of Dice* (Lee et al., 2020; Tarr et al., 2006) uses a dynamic sampler. The task scenario is about purchasing dice from three different companies, some of which seem to be selling poor-quality dice. Participants must use the Law of Large Numbers to find, with confidence, which companies are selling poor-quality dice (see *Quick Links for Activities* section for details).

Implications

Because probability is a tool used by statistics (Franklin et al., 2007; Franklin & Garfield, 2006) probabilistic content is foundationally critical for preservice teachers to interact with regularly. Unfortunately, the word "probability" is only mentioned in one substandard in the CAEP K-6 Elementary Teacher Preparation standards (2018), a narrow prescription for teacher educators to expose teachers to probabilistic concepts. Therefore, teacher educators must find creative pedagogical techniques to intertwine probabilistic concepts *across mathematical domains,* including (but not limited to) the activities described in this paper.

Quick Links for Activities

Investigating the Fairness of Dice: There are three CODAP samplers to decide the fairness of dice produced with the context of companies.

Dice R' Us:
https://codap.concord.org/releases/latest/static/dg/en/cert/index.html#shared=53216
Pips & Dots:
https://codap.concord.org/releases/latest/static/dg/en/cert/index.html#shared=53236
High Rollers:
https://codap.concord.org/releases/latest/static/dg/en/cert/index.html#shared=53237

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IMPACT OF A SUMMER MATHEMATICS ACADEMY ON STUDENTS' EARLY NUMBER SENSE: A TWO-YEAR CASE STUDY

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Early number sense plays an important role in later mathematical development. Students who enter Kindergarten lacking experience with early number concepts may be at risk for falling behind in future mathematics courses. In this qualitative case study, we examined the impact of a summer mathematics academy targeting PreKindergarten and Kindergarten students' understanding of early number concepts. These students, from low SES backgrounds, were identified by the local school district to participate in the academy, as they struggled with early number concepts. Results from this study suggest that the academy as an intervention was helpful in students' development of these concepts.

Introduction

In an effort to support early number sense development in children from underserved populations, a school of education at a mid-sized university partnering with a local school district offers a 4-week summer mathematics academy. The goal of this learning experience is to develop numerical fluency for students aged 4-6 (grades Pre-Kindergarten and Kindergarten). The targeted population includes students from low socioeconomic status (SES) backgrounds. The academy is designed to provide a summer intervention with various checkpoints throughout the subsequent school year. Further, preservice teachers and graduate students from the university, as well as teachers from the local school district, are involved in various planning and instructor roles related to the academy. The academy is a 4-week experience, 4-days a week, for 3 hours each morning at a local museum complex on the university campus. Each day, the children participate in activities targeted to build their understanding of early number concepts. Lead Teachers, selected by university teacher education faculty and local school district leaders, coordinate the instructional experiences. Teacher Assistants, preservice teachers taking education courses at the university, work with the Lead Teachers to provide small group instruction. Students can participate in the academy in the summers prior to Kindergarten and First Grade.

Objectives of the Study

Early childhood interventions can prepare students to enter school with a foundation of number sense and narrow the achievement gap between students from low SES backgrounds and their peers. We designed a summer mathematics academy to support PreKindergarten and Kindergarten students from low SES populations. To determine the impact of the academy on these students who attended for two consecutive summers, we crafted the following research question to guide the study: *In what ways does a summer mathematics academy for early learners impact students' number sense?*

Related Literature

Kindergarten readiness is vital for students' future mathematical achievement. Trajectories for mathematical achievement are established in the early grades, as children who are behind their peers in mathematical knowledge at the start of elementary school tend to fall further behind in subsequent years (Duncan et al., 2007; Lewis Presser, Clements, Ginsburg, & Ertle, 2015). Lacking a strong foundation in mathematics in the early grades may hinder students' later performance, as researchers suggest early mathematical knowledge is a predictor of future mathematical achievement (Claessens & Engel, 2013; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Lynch & Kim, 2017).

Early number sense is a key concept for children. The National Council of Teachers of Mathematics (NCTM; NCTM, 1989) defined number sense as "intuition about numbers that is drawn from all varied meanings of number" (p. 39). A strong sense of number when starting Kindergarten is a predictor of later achievement in advanced mathematics (Duncan et al., 2007; Galindo & Sonnenschein, 2015; Jordan, Glutting, Ramineni, & Watkins, 2010). This foundation of number sense allows students to make connections and fosters a deeper understanding required to make sense of more advanced mathematical topics (Duncan et al., 2007; Jordan et al., 2010).

Improving number sense for low SES children should be a priority in early grades (Jordan et al., 2009), and interventions are a suggested way to target this learning. Interventions have had positive effects on young children's mathematical achievement (Dyson, Jordan, Beliakoff, & Hassinger-Das, 2015; Vennberg & Norqvist, 2018). While school-based interventions are well-documented, there is a need for additional research related to early childhood interventions during summer months. Through a summer intervention focused on early number sense, early learners' mathematics learning trajectories can be supported to positively impact their future mathematical achievement.

Description of the Academy

Taking together the importance of Kindergarten readiness, early number sense, and summer interventions, the researchers designed the academy in the current study. Students were selected for participation by their local school districts, attended in the summer between PreKindergarten and

Kindergarten, and were invited to return for a second consecutive summer. Lead Teachers were responsible for planning and conducting whole group lessons, while Teacher Assistants led small group instruction. The whole group lessons, stations, and small group lessons included literature connections along with art, manipulatives, and games to support the development of early number sense. The lessons and stations encouraged subitizing, composing numbers, using number lines, and the concept of more or less. Further, the skills addressed in the academy aligned with the progress monitoring tool, The Texas Early Mathematics Inventory (TEMI). For a full description of the academy, participant selection, the instructional experiences, and the assessments utilized as well as the roles played by the Lead Teacher and Teacher Assistants, see Kerschen, Cooper, Shelton, and Scott (2018) and Shelton, Kerschen, and Cooper (2020).

Methodology

Participants, who were initially recruited with help from the local school district, included 17 children (eight males, nine females) who attended PreKindergarten at the same local public school prior to their participation in the academy. From these 17 students, 3 students were selected to serve as cases in the current study using criterion-based purposeful sampling for an in-depth qualitative analysis. The selection criteria included: (1) attendance in two consecutive summers of the academy and (2) students from lower-performing small groups in this study.

Qualitative data was collected from daily, electronic reflections from the Teacher Assistants based on their interactions with the students in their small groups and weekly reflections based on the students' progress over the course of each week. The coding structure for the qualitative data analysis was developed by aligning the early number concepts with the TEMI early number categories of Magnitude Comparison, Number Identification, Number Sequences, Quantity Recognition, Place Value, and Addition/Subtraction Combinations. To begin the analysis, one case was randomly selected to be independently coded by members of the research team. To establish uniformity, the researchers met to discuss the coded case and confirming/disconfirming evidence for the codes, which were based on the early number concepts identified by the TEMI. Once there was 100% agreement of codes, the researchers coded the remaining two cases.

Results

The researchers conducted data analysis focusing on three students in the academy, Marcela, Ruth, and Daniella (pseudonyms). Each of the three cases are described in the following sections and include reflections from the Teacher Assistants, who selected the areas to report on concerning the students in their small groups based on the activities or tasks each day.

Marcela

In the first year she attended the academy, Marcela demonstrated growth in Number Identification, Number Sequence, and Quantity Recognition on the TEMI. At the beginning of the academy, Marcela was assessed in Number Identification, specifically in the area of one-to-one correspondence. She was asked to place counters in specified areas on a sheet on a piece of paper. She was told that there was to be one flower per vase, where the counters represented the flowers and the vases were shown on the paper. She was given eight counters to determine if she could place one counter on each vase to signify that each vase had a single flower inside of it. When Marcela was provided with eight counters, she was able to place one counter on each vase. To further assess her understanding of one-to-one correspondence, Marcela was asked to repeat different iterations of the task, given either too many counters or too few counters. When she was given nine counters, she put two counters in one vase. When given seven counters, she kept moving the counters around and did not indicate that there were not enough counters for each vase to have one flower. This revealed that Marcela's understanding of one-to-one correspondence was still developing at this time.

Throughout the academy, the Teacher Assistant provided several reflections on Marcela's growth in this area as they worked on particular tasks as part of the academy intervention. For example, in week 2 while working on a particular task with counters, the Teacher Assistant noticed Marcela had trouble counting more than 4 counters. When asked to count them, Marcela's finger would move from counter to counter faster than her verbal counting.

At the end of the academy, Marcela was provided the same task she was given in the first week, to place counters in specified areas on a sheet on a piece of paper. She was given eight counters to determine if she could place one counter on each vase to signify that each vase had a flower inside of it. She was able to place one counter in each vase when given eight counters, and she was able to explain that there were extra counters when she was given nine, and she was able to explain that she did not have enough counters when she was given seven. This revealed Marcela's growth in the area of Number Identification, specifically one-to-one correspondence, after participating in the academy intervention.

In the second year of the academy, Marcela demonstrated growth in Magnitude Comparison, Place Value, and Addition/Subtraction Combinations on the TEMI. In week 1 Marcela had difficulty identifying key parts of word problems focusing on addition and subtraction. However, Marcela was able to successfully model addition problems using connecting cubes. By week 3, Marcela understood the identity property of addition, recognizing that when 0 is added to a number, the number remains the same. At the end of the third week, the Teacher Assistant summarized Marcela's progress in Addition/Subtraction Combinations in the following way,

> Marcela has improved a lot with adding and subtracting 0 and [adding] 1. At first, she would guess a lot, but towards the end of this week she was really understanding the concept and did not rely on the connecting cubes. She would just automatically count up or down or know the number remained unchanged if we were adding or taking away 0.

While Marcela was becoming comfortable with adding or subtracting 0 and adding 1 to a given number without using the number line, she still seemed to struggle with subtracting 1. The Teacher Assistant noted that Marcela would sometimes forget to subtract 1; rather, she would add it. When instructed to look again at the problem, she typically corrected her error. It seems that in working with a Teacher Assistant during the academy intervention, Marcela showed improvement in the area of Addition/Subtraction Combinations.

Ruth

In the first year of the academy, Ruth demonstrated growth in Magnitude Comparison and Number Identification on the TEMI. At the beginning of the academy, Ruth was assessed in Quantity Recognition, specifically in the area of part/part/whole. She was first asked to count out eight two-color counters. After Ruth demonstrated eight counters, she was asked to show the eight counters in another way. When asked to show eight counters, Ruth was able to count eight counters, but she could not show them a different way. This revealed that Ruth's understanding of part/part/whole was still developing at this time.

Early in the academy, based on the notes from the Teacher Assistant, Ruth grew in her understanding of part/part/whole. For example, in an activity with unifix cubes, students were provided seven cubes and asked to remove one and set it to one side and put the other six to the other side. When Ruth was asked how many cubes they had altogether, she was able to say seven without hesitation. The Teacher Assistant made this observation at the end of week 2,

> I have noticed that my students [including Ruth] are beginning to understand part/part/whole. When we do the activity with the unifix cubes they are able to tell me what they see and what the number is when I ask them how many cubes, they have in all, they still take some time to think, but I can really see that they are understanding the concept better than they did at the beginning of the week.

By the end of week 3, the Teacher Assistant was able to indicate that Ruth was able to use two dice to indicate numbers in different ways. Ruth could use two and five or three and four to show seven. At the end of the academy, Ruth was provided the same task to count eight counters and then show them another way. She was able to count eight counters as well as show the quantity in different ways. This revealed Ruth's growth in understanding part/part whole over the course of the academy.

In the second year of the academy, Ruth demonstrated growth in Number Sequences and Place Value on the TEMI. In the first week, the Teacher Assistant noticed that Ruth was able to consistently, quickly, and correctly count and group items in groups of five. Ruth could count out groups of five and then use these groups and circling the groups to help with unitizing, or the process of counting by fives in this case. By the end of the academy, the Teacher Assistant observed that Ruth was able to complete this same process using groups of ten rather than groups of five. The Teacher Assistant explained, "She was able to group the objects in groupings of ten easily and count by tens. She did this mostly independently - even during group work."

Daniella

In the first year of the academy, Daniella demonstrated growth in Magnitude Comparison and Number Sequences on the TEMI. At the beginning of the academy, Daniella was assessed in areas related to Number Sequences, specifically counting on and counting down. A small set of counters was placed in front of Daniella, and she was asked to count how many there were. Then, one more counter was placed in front of her, and Daniella was asked how many counters there were. Daniella was able to count out a correct number of counters, but she was not able to indicate the correct number of counters when adding or taking away counters without recounting. This revealed that Daniella's understanding of counting on and counting down was not developed.

The Teacher Assistant indicated that Daniella made progress in this area, especially in her ability to count backwards. By week 2, the Teacher Assistant explained, "Daniella has started to be able to count backwards when she sees the numbers in front of her." By week 3, the Teacher Assistant explained how one day when the intervention lesson was completed very quickly, she showed Daniella how to "work backwards to finish a 3 number sequence" when given the last number and after modeling it for her once, she was able to do it with several different numbers.

At the end of the academy, Daniella was provided the same task to identify how many counters were placed in front of her and then determine how many there were without recounting. She was able to count eight counters and indicate how many counters were present when counters were added or taken away without recounting. This revealed Daniella's growth in Number Sequences over the course of the academy.

In the second year of the academy, Daniella again demonstrated growth in Number Sequences on the TEMI. In the first week, Daniella struggled with Addition/Subtraction

Combinations. The Teacher Assistant explained it in this way, "She can tell me that 3+2 is 5, but when it comes to actually working it out on paper, she struggles," recommending that more experience with manipulatives and working these problems out on paper and pencil might be beneficial for Daniella. By the end of week 3, the Teacher Assistant noted Daniella's progress, explaining, "We also worked on +0 and +1 facts and Daniella caught on to the concept fairly well." Later in that same week, the Teacher Assistant provided support for Daniella through the use of unifix cubes and number lines to model subtracting 0 and 1.

In the last week of the academy, Daniella seemed to continue improving in Addition/ Subtraction Combinations. The Teacher Assistant noted that when subtracting numbers from themselves, Daniella understood that the answer would always be zero. Further, "Daniella did a great job today and I think [she] understood the concept of plus or minus 0 and 1. She got all of the independent practice problems right in the given time." Daniella seemed to be able to translate her understanding of Addition/Subtraction Combinations to setting up word problems modeling these. The Teacher Assistant noted on her final observation that, "Daniella can add and subtract 0 and 1 fairly well and she does very good on word problems." While occasionally Daniella had issues with Addition/Subtraction Combinations with values greater than 1, she was still able to model the number sentences correctly using manipulatives.

Discussion and Implications

The research question that guided the study was, *In what ways does a summer mathematics academy for early learners impact students' number sense?* Data analysis suggests that the academy supported students in developing their understanding of critical early number concepts as identified by the TEMI and in reflections from the Teacher Assistants. The students who attended were identified as having gaps in their understanding of mathematics prior to Kindergarten. The use of manipulatives and small group instruction as an intervention during the summer months appeared to help these early learners strengthen their understanding of early number, which, in turn, supported their Kindergarten readiness. Specifically, participating in the academy for two consecutive summers seemed to impact students' understanding in the early number areas of Number Identification, Number Sequences, Quantity Recognition, Place Value, and Addition/Subtraction Combinations as evidenced by the cases.

It may also be important to note that students perhaps could have experienced summer learning loss had they not attended for two consecutive summers. Additional quantitative data was also collected in conjunction with the qualitative data reported in this study to examine students' understanding of early number in comparison with their peers in the academy as an intervention as well as peers who did not attend. These related data and findings will be reported in future papers.

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THE ALIGNMENT OF TEACHER CREATED ONLINE CURRICULA TO MATHEMATICAL MODELING

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Mathematical modeling has garnered national and international interest due its importance. Mathematical modeling can engage students and develop their mathematical understandings, teamwork and communication practices. The ability for teachers to select appropriate mathematical modeling tasks is crucial in ensuring the realization of the aforementioned benefits. The purpose of this study was to investigate the alignment to mathematical modeling of teacher created activities available through the Teachers Pay Teachers (TPT) website. The TPT website, where teachers can buy and sell resources, is used by millions of teachers each year. Through this research we sought to investigate if the curricula that resulted in searches for mathematical modeling at the middle school level is aligned to mathematical modeling. This research contributes valuable information as the prevalence of online resources increases.

Keywords: Mathematical Modeling; Online curricula; Teachers Pay Teachers

Introduction

In recent years, teachers have increasingly made use of online resources to incorporate into their teaching. For example, Teachers Pay Teachers (TPT), an online marketplace for original educational resources that can be bought or sold, had five million educators use their website in the last year (TPT, 2019). While most teachers are looking online for curriculum resources (PBS & Grunwald Associates, 2011), we know very little about how teachers are accessing and selecting these resources for use in mathematics lessons. An important consideration in the curricular resources that are selected is if they truly align with the advertised topics and standards. If teachers have misconceptions about topics this can create issues, as in past research it has been found that teachers have misconceptions about mathematical modeling or incomplete ideas (Anhalt and Cortez, 2015; Gould, 2013; Stohlmann, 2019a). The intellectual rigor, coherence, and appropriateness of materials used in classroom can have a large influence on student learning (Ball & Cohen, 1996; Moore et al., 2013). This is important because for the benefits associated with mathematical modeling to be realized teachers need to have a clear conception of mathematical modeling (Stohlmann, 2019b). Mathematical modeling can develop students' mathematical understanding through different representations, develop 21st century competencies, improve students' attitudes towards mathematics, and increase students' engagement (Stohlmann, 2018). Because of this it is important to analyze the alignment to mathematical modeling of teacher created curricula.

Objectives of the Study

The purpose of this study was to determine the alignment to mathematical modeling of teacher created resources that result in searches for mathematical modeling. We sought to investigate if the activities were truly mathematical modeling activities and also to investigate what teachers perceive as mathematical modeling. With this in mind the research question for this study is the following: *What is the alignment to mathematical modeling of teacher created activities, that result from searches for mathematical modeling, available on the Teachers Pay Teachers website for 8th grade?*

Related Literature

Teachers' Use of Online Curricula

There is limited research on how teachers select online curriculum resources, but the available research findings provide some insight into how teachers select from online-available curricula. Resources that are aligned to standards is an important consideration for teachers. A qualitative study that made use of interviews explored the perceptions of twelve teachers regarding the use of TPT and Pinterest, a social media website. The teachers in the study stated that they search activities by objective, and assess quality of an activity by how well it meets the expectations of the standards to be taught. Further, the teachers liked that they have more access to teacher resources from teachers in a similar school or grade level (Irvine, 2015).

An issue with the availability of large amounts of online resources is how to determine the quality of the resources. Clements and Pawlowski (2012) surveyed users of online educational resources on issues of re-use, quality, and trust. They found that instruments such as peer reviews and rankings could improve the quality of resources from the point of view of teachers. However, online peer review is influenced by many variables (Morrison, 2010). A teacher's beliefs about specific pedagogies can hinder accurate evaluations of teaching resources (Remillard, 2012). The prior knowledge of a person giving the rating will affect the rating results, even if clear criteria or rubrics are provided (Tillema, 2009). This can mislead a teacher to form an incorrect and negative view of a high-quality teaching resource based on a reported negative view of the associated pedagogy of the resource. Additionally, it is difficult to find consensus amongst educators on what constitutes a high quality teaching resource (Sumner et al., 2003). On the TPT website, members can rate and provide comments on the resources. Members that upload their own activities provide the descriptions and can also tag the activities by subject, grade level, and standards. A study done on the TPT website found that the number of ratings and comments rather than the content of the ratings and comments were highly positively correlated with the sales of resources. The mere presence of ratings and comments, rather than the positive versus negative nature of these comments, appeared to influence purchases (Abramovich & Schunn, 2012).

Mathematical Modeling Definition and Related Research

In past research, it has been found that teachers have misconceptions about mathematical modeling or incomplete ideas (Anhalt and Cortez, 2015; Gould, 2013; Stohlmann, 2019a). These misconceptions included mathematical modeling just involving representations, involving unrealistic scenarios, and always resulting in an exact answer (Gould, 2013; Stohlmann, 2019a). Mathematical modeling has also been incorrectly viewed as a teacher demonstration (Anhalt & Cortez, 2015).

There are several different interpretations of mathematical modeling (Kaiser & Sriraman, 2006). Our perspective is aligned with the Contextual perspective which has subject-related and psychological goals. Julie and Mudaly (2007) describe there are two perspectives about teaching and learning mathematical modeling: modeling as content and modeling as a vehicle. Modeling as content focuses more on the technical side of modeling in teaching the modeling cycle, modeling abilities and competencies. In modeling as a vehicle, mathematical modeling activities are considered to be meaningful problem-solving situations to teach mathematics. This perspective is more aligned with having to meet content standards in teaching through modeling, but both perspectives are valuable. Our definition is aligned with modeling as a vehicle which is the perspective adopted by most U.S. teachers given that certain mathematical standards are to be met. In particular our definition of mathematical modeling is that "mathematical modeling is an iterative process that involves open-ended, real world, practical problems that students make sense of with mathematics using assumptions, approximations, and multiple representations" (Stohlmann et al., 2016, p.12). This definition is aligned with the one expressed in the U.S. Common Core State Standards for Mathematics (CCSSM, 2010).

In this study we focused specifically on two key aspects of modeling: using mathematics to make sense of real-world problems and for the problems to be open-ended. We recognize that other aspects are involved in mathematical modeling such as the modeling process and modeling competencies, but we chose to focus on the two key criteria of open-ended and real-world as a basic mathematical modeling definition in order to do the coding of the curricula.

Methodology

In this study we used a deductive coding strategy using a list of preset codes (Corbin & Strauss, 2008). We purposefully sampled the curricula labeled mathematical modeling in two ways. First, we entered the search term, mathematical modeling, and selected 8th grade. We selected the top 40 relevant search results. This was done because it has been found that in recent years the first page of search results on google capture 71% to 92% of search traffic (Shelton, 2017). On TPT, the first page includes 24 items. In order to have a better sample size we increased this to 40 with the rationale that many users would not go past partway through the second page of results. The second form of purposeful sampling was done by selecting 8th grade and those curricula tagged with the 4th Standard for Mathematical Practice (SMP) in the U.S. CCSSM, model with mathematics.

Data Analysis

The data analysis involved two main parts. First, we coded the curricula based on the two criteria discussed above: open-ended and real-world. Some of the resources on TPT are a collection of problems or activities. If any of the included problems were open-ended or real-world, then the resource was coded positively. Those curricula that were coded open-ended and real-world were listed as mathematical modeling activities. Secondly, for those curricula that were not listed as mathematical modeling, we categorized them with an initial set of codes developed from the literature. The following codes were used based on previous incorrect ideas about mathematical modeling: representations, just a real-world problem (not open-ended), and a resource for a teacher to demonstrate how to solve problems. In coding the curricula that were not mathematical modeling, other codes emerged due to some of the curricula not fitting into our initial coding scheme. These will be shown in the results. The Cohen's K coefficient of inter-rater agreement was 0.83, and thus within an acceptable range (Fleiss, 1981; Landis & Koch, 1977). Once coding differences were identified, the raters came to agreement on the discrepancies so that full agreement was reached.

Results and Discussion

8th grade mathematical modeling search

There were 4 (10%) curricula resources that were coded open-ended and 32 (80%) that were coded real-world. The four activities that were coded mathematical modeling involved planning a vacation with a budget, a solar oven design project, designing a floor plan of a house along with calculating the construction costs, and investigating bridge strength in relation to thickness. Representations was the most common misconception that occurred (Table 1). The curricula could be coded as more than one characterization. For example, a resource to help teachers demonstrate how to use the number line to graph inequalities was coded as representations and teacher demonstration. There were six occurrences of science activities that included scale models of the solar system, natural selection and adaptation, a waves lab, evolution, and a wildlife mark and recapture population estimate activity. The game was a jeopardy review game.

Table 1

Characterization of non-mathematical modeling curricula 8th grade search

Characterization	Number of occurrences
Representations (manipulatives, graphs, pictorial or visual representations, and/or real world)	15 (37.5%)
Just real-world problem(s)	7 (17.5%)
Mathematics used in science investigation or activity	6 (15%)
Symbolic practice problems	4 (10%)
Teacher demonstration	2 (5%)
Help students understand or develop the Standards for Mathematical Practice	1 (2.5%)
Game	1 (2.5%)

8th grade SMP 4 search

There were 4 (10%) curricula resources that were coded open-ended and 20 (50%) that were coded real-world. The four activities that were coded mathematical modeling involved creating an idea for a food truck and developing a financial business plan, a project involving purchasing a car and investigating career options, planning a thanksgiving dinner, and investigating the costs and benefits of going environmentally green. Representations was the most common misconception (Table 2). The games were mostly matching card games. The science activity involved graphing and analyzing data from a science experiment.

Table 2

Characterization of non-mathematical modeling curricula 8th grade tagged SMP 4

Characterization	Number of occurrences
Representations (manipulatives, graphs, pictorial or visual representations, and/or real world)	32 (80%)
Game	10 (25%)
Just real-world problem(s)	5 (12.5%)
Help students understand or develop the Standards for Mathematical Practice	2 (5%)
Mathematics used in science investigation or activity	1 (2.5%)
Resource to get to know a graphing calculator	1 (2.5%)

Implications

We found that only 10% of the curricula in our searches could be considered mathematical modeling. The small amount of curricular resources that could be considered mathematical modeling is a concerning issue. When teachers search for online activities, standards are an important consideration (Irvine, 2015). Based on our searches, the curricula that are being uploaded to TPT are not being coded or described accurately in regards to mathematical modeling. This can cause further misconceptions about mathematical modeling. This study adds to the list of misconceptions to include games and mathematics used in the context of science.

Niss (1987) noted that in the past the inclusion of mathematical modeling in different countries happened in a way that was far from uniform. There was and is still today considerable diversity in how modeling is included. Online resources can lead to greater collaboration with teachers from different parts of the world and allow them to learn more about applicable contexts for mathematics. Our study has shown the need for resources to be appropriately described and checked. Mathematical modeling research highlights the importance of how experts see and interpret things differently (Lesh & Zawojeski, 2007). The TPT website does allow for content creators to post information about their experience and expertise. However, those looking for resources may not take the time to look at this information. Based on our research, there is a need for those with expertise in mathematical modeling to make classroom-tested mathematical modeling resources available online for teachers who seek out curricular resources. In order to have the greatest impact on student learning, teachers need support for implementation of mathematical modeling, as well. Identifying appropriate resources to be used in the classroom is only one consideration for an effective lesson.

Further research is needed in regards to how teachers make use of and select online resources. The influence of these resources is important because it could potentially harm student learning, or it could have a positive impact on student learning, and furthermore, use of these resources may need to be more closely considered (Abramovich et al., 2013). The results of this study and prior research on teachers' understanding of mathematical modeling demonstrate the need for professional development on mathematical modeling. Professional development can help teachers develop a clearer understanding of mathematical modeling (Stohlmann, Maiorca, & Olson, 2015). It can also help teachers identify appropriate resources (Higgins & Spitulnik, 2008).

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