# SSMA 2016

## Proceedings — Volume 3


**115th Annual Convention**
October 20 - 22, 2016
Phoenix, Arizona

**Margaret J. Mohr-Schroeder**
**Jonathan Thomas**
Editors

# PROCEEDINGS OF THE 115TH ANNUAL CONVENTION OF THE SCHOOL SCIENCE AND MATHEMATICS ASSOCIATION

**STEAM Rising in Phoenix**

**October 20 - 22, 2016**

**Phoenix, Arizona, USA**

Editors: Margaret J. Mohr-Schroeder & Jonathan N. Thomas

Cite Proceedings Volume as:
Mohr-Schroeder, M. J., & Thomas, J. N. (Eds.). (2016). *Proceedings of the 115th annual convention of the School Science and Mathematics Association* (Vol. 2). Oklahoma City, OK: SSMA.

Cite Proceedings Chapter as:
Author. (2015). Title of chapter. In M. J. Mohr-Schroeder, & J. N. Thomas (Eds.), *Proceedings of the 114th annual convention of the School Science and Mathematics Association* (Vol. 3). Phoenix, AZ: SSMA.

Website: www.ssma.org
The proceedings are also available via http://www.ssma.org/convention-proceedings



ISBN: 978-1-5323-2243-3

# SSMA 2016 ANNUAL CONVENTION: PHOENIX, ARIZONA

*STEAM RISING IN PHOENIX*

## 2016 Program Chairs and Local Arrangements

Ron Zambo, Arizona State University
Juliana Utley, Oklahoma State University
Adrienne Redmond-Sanogo, Oklahoma State University

## Convention Chairs

Melanie Shores, University of Alabama Birmingham
Tommy Smith, University of Alabama Birmingham

## SSMA Leadership

**President, 2014-2016**
Gilbert Naizer, Texas A&M University-Commerce

**President-Elect, 2015-2016**
Stacy Reeder, University of Oklahoma

**Directors-at-Large, 2013-2016**
Timothy Laubach, University of Oklahoma
Ron Zambo, Arizona State University

**Directors-at-Large, 2014-2017**
Charles Emenaker, University of Cincinnati Blue Ash
Elaine Tuft, Utah Valley University

**Directors-at-Large, 2015-2018**
Ken Miller, Montana State University Billings
Margaret Mohr-Schroeder, University of Kentucky

**Co-Executive Directors, 2014-2019**
Melanie Shores, University of Alabama Birmingham
Tommy Smith, University of Alabama Birmingham

**Journal Editor and Office, 2011-2021**
Carla Johnson, Purdue University
Shelly Harkness, University of Cincinnati

**Newsletter Editor, 2013-2016**
Georgia Cobb, University of Montana

**Proceedings Editors**
Margaret Mohr-Schroeder, University of Kentucky
Jonathan Thomas, University of Kentucky

**Special thanks to**

Maureen Cavalcanti and Emma Chadd, University of Kentucky Graduate Students, for all of their hard work on the proceedings.

# School Science and Mathematics Association

## Founded in 1901

The School Science and Mathematics Association [SSMA] is an inclusive professional community of researchers and teachers who promote research, scholarship, and practice that improves school science and mathematics and advances the integration of science and mathematics.

SSMA began in 1901 but has undergone several name changes over the years. The Association, which began in Chicago, was first named the Central Association of Physics Teachers with C. H. Smith named as President. In 1902, the Association became the Central Association of Science and Mathematics Teachers (CASMT) and C. H. Smith continued as President. July 18, 1928 marked the formal incorporation of CASMT in the State of Illinois. On December 8, 1970, the Association changed its name to School Science and Mathematics Association. Now the organizational name aligned with the title of the journal and embraced the national and international status the organization had managed for many years. Throughout its entire history, the Association has served as a sounding board and enabler for numerous related organizations (e.g., Pennsylvania Science Teachers Association and the National Council of Teachers of Mathematics).

SSMA focuses on promoting research-based innovations related to K-16 teacher preparation and continued professional enhancement in science and mathematics. Target audiences include higher education faculty members, K-16 school leaders and K-16 classroom teachers.

Four goals define the activities and products of the School Science and Mathematics Association:

- Building and sustaining a community of teachers, researchers, scientists, and mathematicians
- Advancing knowledge through research in science and mathematics education and their integration
- Informing practice through the dissemination of scholarly works in and across science and mathematics

  Influencing policy in science and mathematics education at local, state, and national level

Mohr-Schroeder, M. J., & Thomas, J. N. (Eds.). (2016). *Proceedings of the 115th annual convention of the School Science and Mathematics Association.* Phoenix, AZ: SSMA.

# PREFACE

These proceedings are a written record of some of the research and instructional innovations presented at the 116th Annual Meeting of the School Science and Mathematics Association held in Phoenix, Arizona, October 20 - 22, 2016. The theme for the conference is *STEAM Rising in Phoenix*.

The blinded, peer reviewed proceedings includes 13 papers regarding instructional innovations and research. The acceptance rate for the proceedings was 72%.

We would like to thank Maureen Cavalcanti and Emma Chadd for their dedication to the technical details of putting together this document. We are pleased to present these Proceedings as an important resource for the mathematics, science, and STEM education community.

Margaret J. Mohr-Schroeder
Jonathan N. Thomas
Co-Editors

# TABLE OF CONTENTS

## VOLUME 3

# USING THE REFORMED TEACHING OBSERVATION PROTOCOL FOR NOVICE TEACHERS

Faye Bruun
Texas A&M University - Corpus Christi
faye.bruun@tamucc.edu

Kim Moore
Texas A&M University - Corpus Christi
kim.moore@tamucc.edu

Joe Champion
Boise State University
joechampion@boisestate.edu

*This study investigates 13 novice teachers who recently graduated from a newly redesigned Grades 4-8 math and science teacher certification program. Researchers used the Reformed Teaching Observation Protocol (RTOP) to analyze the teaching of the novice educators. The teachers completed the same undergraduate teacher education program and continued in an induction program where university faculty mentored them during their first two years of teaching. Findings suggest potential ways to better support novice teachers' use of reformed teaching practices.*

## Introduction

Teacher training programs face the daunting task of preparing educators that are ready to teach all children to think, reason, and solve complex tasks. As standards for K-12 education become more rigorous, teacher training programs must rethink how they can best prepare their graduates to meet the demands of ensuring that all students learn. "The more tightly integrated the learning experience of novices, veteran teachers, and university faculty can become, the more powerful the influence on each other's practices and capacity for constant improvement," (Darling-Hammond, 2006, p.185). The best programs recognize that theory and practice need to be interwoven. Pre-service teachers need to see effective teaching practices modeled throughout their field experiences.

Texas A&M University-Corpus Christi is testing an unconventional preservice strategy for modifying the elementary to middle levels STEM certification and teaching pathway through a fellowship program for undergraduate seniors in the College of Education. ETEAMS fellowships include instructional coaching, co-teaching of evidence-based lessons, and targeted workshops to deepen content knowledge. These initiatives are delivered by STEM education faculty and staff to improve the quality of teaching and learning in middle grades classrooms. A federal grant supports the fellowships along with support for graduates during their first two years of classroom teaching. Program staff observe participants' classroom instruction twice a year using the Reformed Teaching Observation Protocol (RTOP).

## Objectives of the Study

One major outcome goal of the ETEAMS project is to increase evidence-based STEM instructional practices at participating schools. Operationalizing that goal has required ongoing development of the mentoring program offered during participants' first two years of teaching. As program activities get put into place, however, the project leadership has continued to seek a clearer picture of the novice teachers' instructional practices.

The research question is: To what extent are ETEAMS graduates (now novice teachers) implementing reformed teaching practices that they have learned as preservice teachers?

**Theoretical Framework and Related Literature**

The ETEAMS program design flows from a Theory of Action including three core hypothesized implementation claims:

- Novice teachers will engage in enriching middle levels STEM teaching activities, increasing their interest and self-efficacy in middle levels STEM teaching
- Collaborations between novice teachers, teachers, and STEM education faculty in teacher-led grades 4-8 instructional reform will be mutually beneficial and productive
- Novice teachers' participation in STEM experiences and STEM teaching reforms will lead to improved STEM instruction for Grades 4-8 students

In *Strategies and Sources of Support for Beginning Teachers of Science and Math*, Friedrichsen, Chval, and Teuscher, (2007) proposed a model of novice teachers' initiating access to support structures based on the realization that their ideal images of teaching do not match the realities of their classrooms. Support structures were identified as people, programs, and internal and external supports such as mentoring. ETEAMS seeks to address each of these support structures.

Partly due to wide differences across teacher preparation program, research on the instructional practices of novice classroom mathematics teachers has tended to be small-scale and qualitative. However, literature on induction and mentoring of beginning teachers, as synthesized by Ingersoll & Strong (2011), shows some empirical support for the claim that assistance for beginning teachers has a positive impact on teacher classroom instructional practices. Stanulis and Floden (2009) examined the effects of novice teachers receiving an existing district induction program compared to receiving intensive mentoring provided through a school/university partnership, which is similar to the ETEAMS program and this study. Using two matched groups of 12 beginning teachers, they found that the group that had University mentoring showed gains from observations of the novice teachers' instructional practices over those receiving district mentoring.

In a case study of six novice teachers, Roehrig, Bohn, Turner, and Pressley (2008) found that beginning teachers, regardless of induction intensity, declined in their use of effective teaching practices over the course of their first year. The Glazerman and colleagues (2010) study was the largest, most ambitious research of an induction program with a randomized controlled trial methodology, and it found that on-the-job development of beginners takes more than one year to make an impact on teacher practices.

In 1995 the Arizona Collaborative for Excellence in the Preparation of Teachers developed the RTOP observation instrument, with subsequent research by Sawada and colleagues (2002) measured significantly enhanced student learning in reformed classrooms. In this context, reform teaching practices are defined as moving from teacher centered and traditional lecture driven classroom to student centered activity-based learning with multiple opportunities for collaboration of students. Proponents of reformed teaching advocate that classes be "taught via the kinds of constructivist, inquiry-based methods advocated by professional organizations and researchers" (MacIsaac & Falconer, 2002, p. 480). Reformed teaching emerged from the principles of effective teaching introduced in 1988 by the American Association for the Advancement of Science's report on the state of science teaching in the American Educational Institutions *Project 2061: Science for all Americans* (AAAS, 1989) and in 2000 by the NCTM *Principles and Standards for School Mathematics*.

## Methodology

### Setting and Participants

The participants in this study were among the first graduates of the ETEAMS program, all of whom are currently full-time classroom teachers in nearby school districts. Nine of these novice teachers remained within the large urban partnership district in which they completed student teaching, including four working at one of the ETEAMS fellowship partner schools (two each at the elementary and middle school levels). The four other teachers work in three outlying smaller school systems. All 13 novice teachers were observed in the fall of their first year as classroom teachers, with 11 of them observed again in the spring.

### Treatment

The participants were either elementary (EC-6) or middle school teachers (6-8) who had been a part of the ETEAMS fellowship program during their final year as an undergraduate. They had participated in the following grant sponsored activities: (a) STEM Thursdays where they collaboratively planned and taught 5E lessons (Bybee, 2014) to Grades 4-8 students, (b) a 30- hour summer authentic research experience where they joined STEM faculty in ongoing laboratory and field investigations, (c) mathematics and/or science workshops focused on deepening content knowledge, and (d) professional development on problems solving and the nature of science.

The mentoring program, includes two years of support including monthly working dinner meetings with STU faculty and staff, classroom-based instructional coaching, and facilitated opportunities to attend and present at statewide mathematics and science conferences. During the monthly dinner meetings, ETEAMS faculty deliver targeted professional development (PD) and then provide time for the novice teachers to share successes and struggles from their work experiences.

### Data Collection and Instruments

The nature of participants' implementation of mathematics and science instructional practices was described using composite scores from classroom observations of the novice teachers by the researchers. Each novice teacher was observed once per semester. Though the RTOP instrument is used by researchers on other projects, the staff observed several of the teachers together to ensure inter-rater reliability. The RTOP was selected as the instrument to measure reformed teaching because it provides a standardized means for detecting the degree to which classroom instruction uses student-centered, engaged learning practice (Lawson et al., 2002; MacIsaac & Falconer, 2002; Sawada et al., 2002). The RTOP instrument is a holistic measure of the presence/absence of specific teaching strategies divided into five subscales: (a) Lesson Design and Implementation, (b) Content: Propositional Knowledge, (c) Content: Procedural Knowledge, (d) Classroom Culture: Communicative Interactions between student-student, and (e) Classroom Culture: Student/teacher Relationship. It is a 25-item classroom observation protocol that is standards based, inquiry oriented, and student centered. Each subscale features 5 items each that are scored on a scale of 0-4, for a maximum possible score of 100 points.

The researchers then divided the 25 RTOP statements into three categories: targeted, supportive, and extraneous. The 11 target statements in the observation protocol were identified as explicit outcomes of the fellowship and mentoring programs. These items were aligned to the 5E instructional model where exploration intentionally precedes explanation. Moreover, in 5E the teacher "becomes a coach with the tasks of listening, observing, and guiding students as they clarify their understanding" (Bybee, 2014, p. 11). As the teacher steps back, the majority of the communication originates with the students. In addition to the pedagogical knowledge, the ETEAMS project seeks to prepare the students to have a deep, conceptual understanding of the mathematics and science content. This knowledge is measured in RTOP items 6, 7, and 8.

**11 “Targeted” RTOP Items**

2) The lesson was designed to engage students as members of a learning community.

3) In this lesson, student exploration preceded formal presentation.

6) The lesson involved fundamental concepts of the subject.

7) The lesson promoted strongly coherent conceptual understanding.

8) The teacher had a solid grasp of the subject matter content inherent in the lesson.

16) Students were involved in the communication of their ideas to others using a variety of means and media.

17) The teacher’s questions triggered divergent modes of thinking.

18) There was a high proportion of student talk and a significant amount of it occurred between and among students.

21) Active participation of students was encouraged and valued.

24) The teacher acted as a resource person, working to support and enhance student investigations.

25) The metaphor “teacher as listener” was very characteristic of this classroom.

In order to support the validity of RTOP scores, the researchers rated each of the participants holistically on an ordinal scale of “developing”, “proficient”, or “accomplished” in their use of reformed teaching practices. These holistic scores were triangulated with the RTOP scores, with RTOP scores further analyzed to compare fall and spring observations as well as to investigate potential differences across grade levels.

**Results and Discussion**

Figure 1 shows how the teachers scored on the RTOP instrument in the fall and spring. The comparison box plots in Figure 1 shows that the participants’ RTOP scores tended to be similar in fall and spring, with lower scores in spring. Figure 2 shows how the novice teachers categorized by holistic ratings scored during both observations by grade level. The individual novice teachers are identified by codes of the form “F00”.

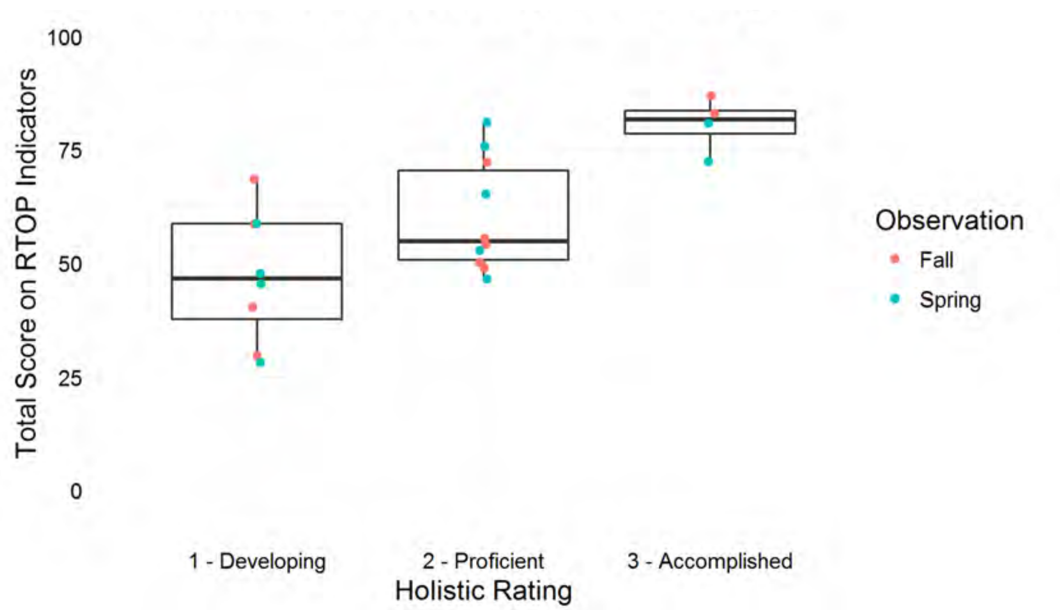



*Figure 1. Distributions of Fall and Spring RTOP Scores by Holistic Ratings*

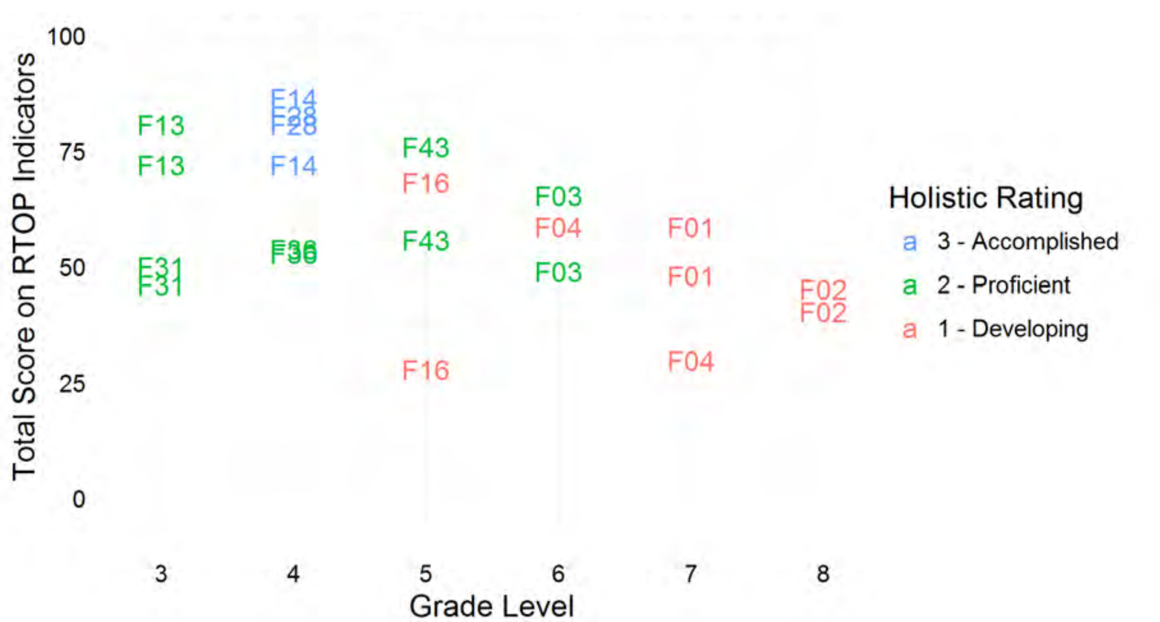



*Figure 2. Overall RTOP Scores by Holistic Rating, Fellow, and Grade Level*

As indicated in Figure 2, the novice teachers in the lower grade levels tended to perform better on reformed teaching practices (on both the RTOP and holistic ratings) than teachers in the middle grade levels. As might be expected, all but one of the participants (F04) taught in the same grade level during both observations.

One important limitation of the results is that, regardless of how valid and reliable the observation instrument, a single, relatively short classroom observation may not be sufficient to accurately characterize an individual's teaching strategies (Glazerman et al., 2010). However, participants' RTOP scores tended to be similar across time points, and also similar to the holistic ratings. "Accomplished" exceeded "proficient" which exceeded "developing" on both the overall and target RTOP scales; this was statistically significant at alpha = .001. That is, the holistic ratings and RTOP scores converged to indicate consistent indications of the novice teachers' use of reformed instructional practices.

## Implications

This small scale exploratory study suggests that some of the reformed practices that the novice teachers learned during their teacher training program translated into practice in their first teaching assignment. However, it is important to keep in mind that many of the participants struggled to fully implement reformed teaching practices in their mathematics classrooms. Marbach-Ad and McGinnis (2009) suggest that beginning educators tend to value the pedagogy strategies of veteran teachers over their own, even when those strategies are more traditional. "Novices struggle to provide change agency within the school environment unless the community within which they work supports their attempts; if not, they succumb to traditional socialization processes," (Allen, 2009). Teachers with internal loci of control tend to maintain their beliefs and practices while those with external loci of control are more likely to emulate the veteran teachers (Cady, Meier, & Lubinski, 2006).

Holistically, some of the practices we saw in observations unfortunately suggested that the novice teachers tended to move toward more traditional, teacher-centered instruction when preparing for standardized testing in the latter half of the school year. This highlights long-standing questions about mentoring new teachers, such as "can an induction program simultaneously promote teachers' skill in engaging students in higher order inquiry, while also promoting teachers' ability to teach standardized test taking, or are these contradictory imperatives calling for completely different induction emphases?" (Ingersoll & Strong, 2009, p. 42). The participating teachers seemed to hold the belief that the best way to prepare students for the high-stakes exams in Spring was to have weeks of lecture-based review with many opportunities to practice test-formatted items. The apparent scarcity of more student-centered models for test preparation was one area of concern.

Another potential area for further research is exploring why middle school teachers displayed less reformed teaching strategies than elementary teachers. In the context of this study, there seems to be more uniformity of curriculum and instruction at the middle school level where novice teachers are given lesson plans by a department chair; they have minimal amounts of decision making power. The elementary schedule is much less rigid than the middle school schedule and generally allows more time if needed for an individual lesson. Perhaps additional, larger scale observations of novice teachers completing reform-oriented elementary and middle grades teacher preparation programs can help clarify the extent to which this pattern extends to other contexts.

## References


Allen, Jeanne M. (2009). Valuing practice over theory: How beginning teachers re-orient their practice in the transition from the university to the workplace. *Teaching and Teacher Education: An International Journal of Research and Studies*, *25*(5), 647-654.

American Association for the Advancement of Science. (1994). *Science for all Americans: Project 2061*. Oxford University Press.

Bybee, R. W. (2014). The BSCS 5E instructional model: Personal reflections and contemporary implications. *Science and Children*, *51*(8), 10-13.

Cady, J., Meier, S., & Lubinski, C. (2006). Developing mathematics teachers: The transition from preservice to experienced teacher. *The Journal of Educational Research, 99*(5), 295-305.

Darling-Hammond, L. et al. (2006). *Powerful teacher education: Lessons from exemplary programs*. San Francisco: Jossey- Bass.

Friedrichsen, P., Chval, K. B., & Teuscher, D. (2007). Strategies and sources of support for beginning teachers of science and mathematics. *School Science and Mathematics*, *107*(5),169-181.

Glazerman, S., Isenberg, E., Dolfin, S., Bleeker, M., Johnson, A., Grider, M., & Jacobus, M. (2010). *Impacts of comprehensive teacher induction: Final results from a randomized controlled study*. NCEE 2010-4027. Washington, DC: U.S. Department of Education.

Ingersoll, R. & Strong, M. (2011). The impact of induction and mentoring programs for beginning teachers: A critical review of the research. *Review of Education Research*, *81*(2), 201-233. doi: 10.3102/0034654311403323

Lawson, A. E., Benford, R., Bloom, I., Carlson, M. P., Falconer, K. F., Hestenes, D. O., Judson, E., Piburn, M. D., Sawada, D., Turley, J., & Wyckoff, S. (2002). Evaluating college science and mathematics instruction: A reform effort that improves teaching skills. *Journal of College Science Teaching*, *31*(6), 388-93.

MacIsaac, D. L. & Falconer, K. A. (2002). Reforming physics education via RTOP. *The Physics Teacher, 40*(8), 479-485.

Marbach-Ad, G., & McGinnis, J. R. (2009). Beginning mathematics teachers' beliefs of subject matter and instructional actions documented over time. *School Science and Mathematics, 109*(6), 338-354.

National Council of Teachers of Mathematics. (2000). *Principles and Standards of School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Roehrig, A. D., Bohn, C. M., Turner, J. E., & Pressley, M. (2008). Mentoring beginning primary teachers for exemplary teaching practices. *Teaching and Teacher Education, 24*(3), 684-702.

Sawada, D., Piburn, M., Judson, E., Turley, J., Falconer, K., Benford, R. & Bloom, I. (2002). Measuring reform practices in science and mathematics classrooms: The Reformed Teaching Observation Protocol. *School Science and Mathematics*, *102*(6), 245-253.

Stanulis, R. N., & Floden, R. E. (2009). Intensive mentoring as a way to help beginning teachers develop balanced instruction. *Journal of Teacher Education, 60*(2), 112-122.


## Acknowledgements


This material is based upon work supported by the National Science Foundation under Grant No. 1321319. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation or the author's institutions.

# VALIDATION OF THE MATHEMATICAL MODELING KNOWLEDGE SCALE (MMKS) WITH PRACTICING TEACHERS

Reuben S. Asempapa
Penn State Harrisburg
rsa26@psu.edu

*This study emphasizes assertions that teachers' mathematical content knowledge plays an important role in their teaching profession. This paper reports on a study to design and empirically measure teachers' knowledge of the nature of mathematical modeling from a Midwestern school district (n = 71). The development of the scale included item generation, experts' reviews, item analysis, and factor analysis. Reliability, factor analysis, and scaling work with the items confirmed the usefulness of the scale, with Cronbach's alpha (α) =.80. Results from the study suggest a psychometrically valid and reliable scale for measuring teachers' knowledge of the nature of mathematical modeling.*

## Introduction

Mathematical modeling strongly influences what mathematics students learn and how they learn it. Researchers and standards emphasize the need to address the skills and understanding of mathematical modeling in the teaching and learning of mathematics (Blum, 2015; Blum & Borromeo Ferri, 2009; Lesh, 2012; National Council of Teachers of Mathematics [NCTM], 2000; National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010; Pollak, 2011). Likewise, mathematical modeling has gained increased focus in assessments for school mathematics—both nationally and internationally (Organisation for Economic Co-operation and Development [OECD], 2003; Partnership for Assessment of Readiness for College and Careers [PARCC], 2014). However, how do teachers conceptualize mathematical modeling and what is their knowledge of the nature of mathematical modeling in school mathematics? Therefore, these issues present an opportunity to design appropriate tools to measure teachers' knowledge related to mathematical modeling.

In this paper, the author first addresses mathematical modeling and teachers' knowledge, noting the effect of content knowledge on teachers' professional competence. Second, the paper discusses the phases used in developing the scale to represent knowledge of the nature of mathematical modeling and describes the methods used to collect data. Third, the paper provides scaling results, and validity and reliability evidence from a pilot study. Finally, the paper concludes with implications for teacher preparation programs and professional development in relation to mathematical modeling education.

## Purpose of the Study

This research is part of a larger study investigating teachers of mathematics knowledge about the nature of mathematical modeling and attitude toward such modeling. Because the Common Core (NGA Center & CCSSO, 2010) and the Next Generation Science Standards (NGSS Lead States, 2013) point to modeling as a central, unifying theme across mathematics and science education, this study sought to investigate the construct knowledge of the nature of mathematical modeling among K–12 teachers of mathematics. Additionally, NCTM's standards emphasize models and mathematical modeling in problem solving (NCTM, 2000, 2009). The practice of mathematical modeling is consistent with the Common Core standards of mathematical practice: model with mathematics and echoes the effective teaching practices and productive dispositions as explained in NCTM's *Principles to Action* (NCTM, 2014).

With the increased importance of mathematical modeling nationally and internationally, it is essential to investigate teachers' knowledge on mathematical modeling. Most K–12 teachers of mathematics have misconceptions about mathematical modeling and the modeling process (Gould, 2013; Spandaw & Zwaneveld, 2010; Wolfe, 2013), and lack knowledge about the nature of mathematical modeling (Blum, 2015). Consequently, this study investigated this dilemma concerning teachers' knowledge about the nature of mathematical modeling. Furthermore, the psychometric properties of the scale were assessed.

## Significance and Related Literature

For the past 40 years, topics that have been central to mathematics education concerned the relationships between mathematics and the real-world. Thus, mathematical modeling is no new phenomenon in the area of mathematics education. The phrase *mathematical modeling* is used to denote any relations whatsoever between mathematics and the real world (Blum & Borromeo Ferri, 2009; Swetz & Hartzler, 1991). Mathematical modeling is more than simply presenting students with traditional word problem. Mathematical modeling provides much more "powerful and effective ways to help students become (a) better problem solvers, and (b) better able to use mathematics in real life situations beyond school" (Lesh, 2012, p. 197). Studies have shown that mathematical modeling supports and motivates students' interest in mathematics (English & Watters, 2004; Pollak 2011). Additionally, when students are provided opportunities to engage in modeling tasks, their engagement reflect improvement in their mathematics achievements (Boaler, 2001). Therefore, it was important to explore teachers of mathematics knowledge of the nature of mathematical modeling.

Shulman (1986) emphasized the importance of teacher's content knowledge as a central aspect of teachers' professional competence. Twenty-two years later, Ball, Thames, and Phelps (2008) explained that teachers of mathematics need certain knowledge domains to teach mathematics effectively, because their knowledge does affect student learning. However, teacher education has been criticized in the area of mathematics content knowledge and in particular, mathematical modeling, for some time without its effectiveness being analyzed empirically (Kaiser, Schwarz, & Tiedmann, 2010). Equally, there is increasing evidence of the limitations in the content knowledge of mathematical modeling among K–12 teachers of mathematics (Blum & Borromeo Ferri, 2009; Spandaw & Zwaneveld, 2010). Therefore, examining teachers' knowledge of the nature of mathematical modeling is timely and cannot be overlooked.

## Methodology

To learn more about the issue, the author first began to write and develop items, and later pilot tested the items intended to represent knowledge of the nature of mathematical modeling. Using several sources, an initial Mathematical Modeling Knowledge Scale (MMKS) was developed with 22 items; formats included true or false items, multiple choice questions and an open-ended question. Using cognitive interviews, reviews with content experts and practicing teachers, as well as item analysis and factor analysis, the initial 22 items on the scale were modified and honed to a 13-item scale. The final scale included 12 true or false items and one open-ended question. The content experts and cognitive interviewers provided inputs as to the relevancy, adequacy, accuracy, and wording of items to establish content validity of the scale. Based on their comments and feedback, items considered irrelevant or redundant were deleted, and others were reworded to improve accuracy or clarity. The study employed a survey research design as described by DeVellis (2012) and Fowler (2014). A descriptive survey research design (cross-sectional) was used and K–12 teachers of mathematics in a Midwestern state were self-selected for this study.

Because of the nature of the study, both purposeful and convenience sampling (Creswell, 2009) were employed to identify the participants who responded to this survey online. The specific form of data collection was a web-based self-administered survey (Creswell, 2009; Dillman, 2007). The main goal of the data analysis for the pilot study was to have a valid and reliable scale that measures teachers of mathematics knowledge about the nature of mathematical modeling. Survey data are only acceptable to the degree to which they are determined valid and reliable (DeVellis, 2012; Fowler, 2014). Hence, statistical procedures used to demonstrate the reliability and validity of the scale in this study included univariate analysis, reliability analysis, and exploratory factor analysis (EFA). In particular, Cronbach's coefficient alpha was utilized in this study to measure the internal consistency reliability of the scale; one of the common forms of reliability mostly used in social science research (Cronbach, 1951). The SPSS statistical software was used for all the analyses. All analyses were considered statistically significant with $p < .05$.

## Results and Discussion

In this analysis, the author answered two main questions: (a) can the items included on the MMKS provide valid and reliable measures of teachers' knowledge of the nature of mathematical modeling? and (b) how do teachers' conceptualize the nature of mathematical modeling? The sample on the scale consisted of 71 teachers of mathematics. Descriptive statistics on the demographics were based on 62 teachers, because not all participants responded to the demographic questions. Of these 62 teachers, 77% of respondents were 35 years or older and almost 60% of the sample were identified as White or Caucasian. The respondents included 36 grades K–5 teachers, nine grades 6–8 teachers, and 17 grades 9–12 teachers of mathematics. Concerning gender, 85% of the sample self-identified as female, and 15% as male. The reliability and factor analysis of the MMKS scores was based on the 12-item true or false questions. Thus, the total possible score on the MMKS was 12. Items answered correctly on the MMKS were coded a score of "1," and items answered incorrectly were coded a score of "0."

The overall total mean score on the MMKS was *10.20* (*SD* = 2.34). An independent sample *t*-test indicated that scores on the scale were statistically significantly higher for female teachers ($M = 10.42, SD = 2.13$) than for male teachers ($M = 8.89, SD = 3.14$), $t(60) = 2.07, p < .05, d = .58$. However, Levene's test indicated unequal variances ($F = 5.26, p = .018$). Cronbach's coefficient alpha was used to calculate the reliability of the MMKS scores. To determine the item reliability, the author examined the correlation matrix between items and the item–total correlations. Although two of the 12 items on the MMKS had item-total correlations less than .30, all 12 items were retained in the analysis because of their correlations ($r \geq .25$) and theoretical relevance (Nunnally & Bernstein, 1994; Osterlind, 2010). The overall internal consistency reliability of the MMKS for this sample was .80, indicating a good reliable scale (DeVellis, 2012; Fowler, 2014). Table 1 provides information on the item–total correlations and alpha values on the MMKS.

In assessing the internal structure of the items on the MMKS, the author used exploratory factor analysis to find out about the interrelationships among the items. The Kaiser-Meyer-Olkin measure of sampling adequacy of .70 was acceptable. In addition, Bartlett's test was statistically significant ($p < .001$) and using the rule of subjects–to–variable ratio of no lower than five (Bryant & Yarnold, 1995); therefore, common factor analysis was appropriate for the data.

Table 1

*Item–Total Correlations of all the 12 Items on the MMKS*

| Items | *M* | *SD* | *SE* | ITC | α |
|---|---|---|---|---|---|
| Item 1 | .92 | 0.28 | .03 | .27 | .80 |
| Item 2 | .90 | 0.30 | .04 | .59 | .77 |
| Item 3 | .80 | 0.40 | .05 | .48 | .78 |
| Item 4 | .72 | 0.45 | .05 | .28 | .80 |
| Item 5 | .86 | 0.35 | .04 | .43 | .79 |
| Item 6 | .86 | 0.35 | .04 | .54 | .78 |
| Item 7 | .77 | 0.42 | .05 | .27 | .80 |
| Item 8 | .90 | 0.30 | .04 | .44 | .79 |
| Item 9 | .82 | 0.39 | .05 | .63 | .77 |
| Item 10 | .94 | 0.23 | .03 | .39 | .79 |
| Item 11 | .93 | 0.26 | .03 | .31 | .80 |
| Item 12 | .77 | 0.42 | .05 | .69 | .76 |

*Note*: $N = 71$; ITC = item–total correlation.

Examining the factor loadings and extracting only one factor based on theoretical relevance to interpret the items explained about 29% of the shared variance on the MMKS for this sample. All the factor loading values were greater than .30, indicating the items correlate well with the whole scale. Thus, using one factor to interpret all the items seems strongest statistically and most interpretable. The single factor extracted on the MMKS was labeled *knowledge on modeling*.

The open-ended question asked teachers of mathematics to express their thoughts or describe exactly the nature of mathematical modeling. This helped in assessing how teachers' conceptualize the nature of mathematical modeling. Specifically, they were asked to write a brief definition for the phrase mathematical modeling. A total of 54 teachers responded to this question. Teachers' responses were read and categorized as poor, fair, good, and excellent by the researcher. The ratings were based on the rubric provided in Figure 1. Because teachers' responses were typed there were no ambiguity about their responses, the statements were clear and straightforward. The ratings were coded as 4 = excellent, 3 = good, 2 = fair, and 1 = poor.

There were notable findings in examining teachers' responses on their knowledge or understanding about the phrase mathematical modeling. Of the 54 teachers who responded to this question, only 7% of the responses could be categorized as excellent responses. Most of the teachers had misconceptions with mathematical modeling and they confused mathematical modeling with modeling mathematics.

| Category | | | |
|---|---|---|---|
| *Excellent = 4* | *Good = 3* | *Fair = 2* | *Poor = 1* |
| Definition<br>demonstrates complete understanding and provides detail explanation. It states almost all steps involved in the modeling process. Links mathematics, real world situations, and the translation between the two. | Definition<br>demonstrates basic understanding and provides minimal explanation. It mentions more steps involved in the modeling process. There is no link between mathematics and the real world. | Definition<br>demonstrates little understanding and little to no explanation. It mentions fewer steps involved in the modeling process. There is no link between mathematics and real world situations. | Definition<br>shows no evidence of understanding of the phrase mathematical modeling. |

*Figure 1. A Rubric for Evaluating the Definition of Mathematical Modeling*

Figure 2 provides the distribution of respondents' responses about the meaning of the phrase mathematical modeling. Most of the teachers' explanation or definition incorrectly assumed mathematical modeling as using physical objects, manipulatives, or representations to solve mathematics problem. Experiences shared by the respondents indicated the phrases *mathematical modeling* and the *modeling process* were a new terminology to most of the teachers, and they have little or no experience with mathematical modeling practices. Most of their explanations failed to recognize mathematical modeling as an iterative process that involves choices and assumptions by the modeler.

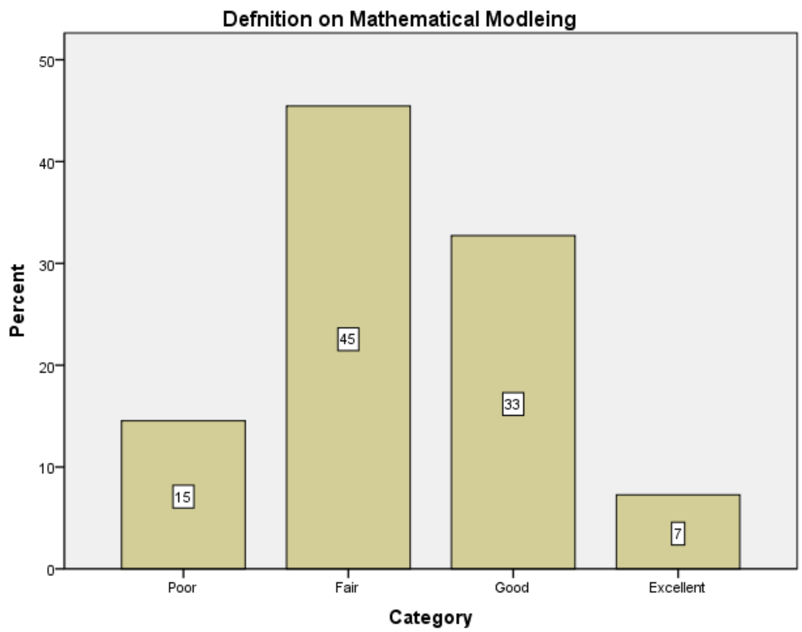



*Figure 2. A bar chart showing teachers' responses about the phrase mathematical modeling*

## Conclusion and Implications

This study's goal was to develop and psychometrically assess a scale that measures practicing teachers of mathematics knowledge of the nature of mathematical modeling. The development and testing of a scale is a complex process. Results from the pilot study revealed that participants had a satisfactory knowledge of the nature of mathematical modeling. However, from their responses on the open-ended question, most of the teachers had misconceptions about the phrase *mathematical modeling*. The reliability and validity evidence provided, as well as the psychometric properties of the MMKS demonstrates its potential in mathematics education research. Therefore, the MMKS can be used as part of ongoing evaluation of teachers' professional knowledge of mathematical modeling education.

Having a scale that assesses teachers' professional knowledge on mathematical modeling will benefit professional development on mathematical modeling. Teachers' professional competence on mathematical modeling will enhance the teaching and learning of mathematics. Results from this study and other published materials (see Kaiser, Schwarz, & Tiedmann, 2010; Spandaw & Zwaneveld, 2010) indicate a need exists for mathematical modeling training standards or courses to be integrated in teacher preparation programs for teachers of mathematics. Finally, the psychometric results suggest the scale is appropriate, and the author hopes the MMKS will benefit mathematics educators, researchers, and teachers, and advance mathematical modeling education in school mathematics.

## References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*, 389–407.

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In S. J. Cho (Ed.), *Proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges* (pp. 73–96). New York, NY: Springer.

Blum, W., & Borromeo Ferri, R. (2009). Mathematical modelling: Can it be taught and learnt? *Journal of Mathematical Modelling and Application*, *1*, 45–58.

Boaler, J. (2001). Mathematical modelling and new theories of learning. *Teaching Mathematics & Its Applications*, *20*, 121–127.

Bryant, F. B., & Yarnold, P. R. (1995). Principal components analysis and exploratory and confirmatory factor analysis. In L. G. Grimm & R. R. Yarnold (Eds.), *Reading and understanding multivariate statistics* (pp. 99–136). Washington, DC: American Psychological Association.

Creswell, J. W. (2009). *Research design: Qualitative, quantitative, and mixed methods approaches*. (3rd ed.). Thousand Oaks, CA: Sage.

Cronbach, L. J. (1951). Coefficient alpha and the internal structure of tests. *Psychometrika,16,* 297–334.

DeVellis, R. F. (2012). *Scale development: Theory and applications*. (3rd ed.). Los Angeles, CA: Sage.

Dillman, D. A. (2007). *Mail and internet surveys: The tailored design method* (2nd ed.). Hoboken, NJ: John Wiley & Sons.

English, L. D., & Watters, J. J. (2004). Mathematical modelling with young children. In J. M. Hoines & B. A. Fuglestad (Eds.), *Proceedings of the 28th International PME Conference* (pp. 335–342). Bergen, Norway: Bergen University College.

Fowler, F. J. (2014). *Survey research methods* (5th ed.). Thousand Oaks, CA: Sage.

Gould, H. T. (2013). *Teachers' conceptions of mathematical modeling*. Retrieved from http://academiccommons.columbia.edu/item/ac:161497

Kaiser, G., Schwarz, B., & Tiedemann, S. (2010). Future teachers' professional knowledge on modeling. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford, (Eds.), *Modeling students' mathematical modeling competencies. ICTMA 13* (pp. 433–444). New York, NY: Springer.

Lesh, R (2012). Research on models & modeling and implications for common core state curriculum standards. In R. Mayes, L. Hatfield, & S. Belbase, (Eds.), *WISDOM$^e$ Monograph: Quantitative reasoning and mathematical modeling*: *A driver for STEM integrated education and teaching in context*, (Vol. 2, pp. 197–203), Laramie, WY: University of Wyoming.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics.* Washington, DC: Author. Retrieved from http://corestandards.org/assets/CCSSI_Math%20Standards.pdf

Next Generation Science Standards Lead States. (2013). *Next generation science standards: For states, by states*. Washington, DC: National Academies Press.

Nunnally, J. C., & Bernstein, I. H. (1994). *Psychometric theory* (3rd ed.). New York, NY: McGraw-Hill.

Organisation for Economic Co-operation and Development (2003). *The PISA 2003 assessment framework–mathematics, reading, science and problem solving, Knowledge and Skills.* Paris, France: OECD Press.

Osterlind, S. J. (2010). *Modern measurement: Theory, principles, and applications of mental appraisal*. Upper Saddle River, NJ: Pearson.

Partnership for Assessment of Readiness for College and Careers. (2014). *The PARCC assessment.* Retrieved from http://www.parcconline.org/parcc-assessment

Pollak, H. O. (2011). What is mathematical modeling? *Journal of Mathematics Education at Teachers College, 2*, 64.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Spandaw, J., & Zwaneveld, B. (2010). *Modelling in mathematics' teachers' professional development*. Paper presented at the Proceedings of the sixth Congress of the European Society for Research in Mathematics Education –Working group 11, (p. 2076–2085), Lyon, France: INRP.

Swetz, F., & Hartzler, J. S. (1991). *Mathematical modeling in the secondary school curriculum: A resource guide for classroom exercises*. Reston, VA: National Council of Teachers of Mathematics.

## Acknowledgements

This work was supported by the Patton College of Education, Ohio University. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the author and do not necessarily reflect the views of the Patton College of Education.

# MORE THAN A STORY: INTEGRATING LITERATURE IN THE MATHEMATICS AND SCIENCE CLASSROOM

Elaine Cerrato Fisher
University of South Florida
cerrato@usf.edu

*Content area literacy is a growing trend across elementary and secondary school curriculum in alignment with the Principles and Standards for School Mathematics (NCTM, 2000) and NGSS Framework for K-12 Science Education (2011). The purpose of this paper is to share ways in which preservice teachers integrated trade books in the elementary mathematics and science classroom to improve literacy skills while introducing and supporting mathematics and science topics. The paper will include examples of content and teaching connections, sample literature books, resources, lesson ideas, and both student and PSTs excerpts sharing their experiences.*

## Introduction

Content area literacy is a growing trend across elementary and secondary school curriculum in alignment with the *Principles and Standards for School Mathematics* (NCTM, 2000) and *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014). Because many mathematical ideas and concepts are abstract or symbolic in nature, children's literature offers teachers the opportunity to present and discuss these ideas and concepts within the context of a story, using illustrations, and more informal, familiar language (Miners & Pascopella, 2007).

This in turn, has the potential to make learning mathematics less intimidating and more engaging, especially for students who demonstrate anxiety related to mathematics (Beane, 1995; Greene, 1991; Hurley, 2001; Tank, 2014). Further, using children's literature for mathematics teaching provides students with additional opportunities, encouragement, and support for reading, writing, listening, and speaking in mathematics classes (Golden, 2012; Thompson, Kersaint, Richards, Hunsader, & Rubenstein, 2008).

Equally important, educators are concerned with how students develop the proficiencies needed to engage in scientific inquiry, including how to read, write, and reason with the language, texts and dispositions of science (Pearson, Moje, & Greenleaf, 2010). The ability to make meaning of oral and written language representations is central to robust science knowledge and full participation in public discourse about science. When reading and writing are cast as tools for investigating phenomena, students can learn how to build on and expand the work of other scientists by reading about the designs and findings of others (Pearson et al., 2010).

## Objectives/Purpose

This paper describes a classroom project for elementary preservice teachers (PSTs), which afforded them the opportunity to explore the integration of content related trade books in their internship classes. The primary objective of the assignment was to provide the PSTs with an array of texts they might incorporate into their mathematics and or science lessons and to find new, innovative ways to support and potentially enhance their instruction. I was also interested in learning the PSTs' and K-5 students' responses regarding their experiences. In what follows, I provide related literature, describe the general structure of this task, details of of the experiences related to PST implementation, and close with implications for supporting teachers in further development of content literacy integration for this assignment.

## Significance and Related Literature

In a time of standards, assessment, and accountability, increased stresses are placed on students to demonstrate an understanding of mathematics and science content and on teachers to assess and determine the depth of student comprehension (Douville, Pugalee, & Wallace, 2003; Rearden & Broemmel, 2008; Pearson et al., 2010). One of the major goals of elementary mathematics and science instruction in this context is the development of mathematical and scientific literacy in K-5 students. Literacy in general, but specifically reading, is central across the content areas. Mathematics and science teachers who integrate literature into the content areas distinguish mathematical and scientific understanding involves reading and writing (Moyer 2000).

Much of the extant literature that argues for literacy integration in content areas recognizes just as learning mathematics and science content can be frightening to students, literacy can be a terrifying word to many science and mathematics teachers (Golden, 2012; Price 2009). To ensure a symbiotic pace in learning in all content areas and in conjunction with the International Reading Association, NCTM (2000) and Thompson et. al., (2008) supports the goal of increasing literacy for all learners.

According to the *National Science Education Standards* (NRC, 1996), "Scientific literacy entails being able to read with understanding articles about science in the popular press and to engage in social conversation about the validity of the conclusions." Comparatively, NCTM's (2014) *Principles to Actions Ensuring: Mathematical Success for All,* describes mathematical literacy as "an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts" and "assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgements and decisions needed by constructive, engaged, and reflective citizens." Both definitions connect literacy as "the ability to read and make sense of written symbols in a variety of settings and subject areas and then be able to locate information, evaluate it critically, synthesize it, and communicate it." (Miners & Pascopella, 2007, p. 29; NCTM, 2014; NRC, 1996).

NCTM's (2000) *Principles and Standards for School Mathematics* emphasized the important role communication plays in helping young children construct knowledge and form links between their informal notions and the abstract symbolism of mathematical ideas (Moyer, 2010). Equally significant, Draper (2002) suggested promoting literacy within the content classroom by creating print-rich environments, explicitly teaching text structure, by addressing precise vocabulary, and combining purposeful discourse. Through these books students see mathematics in a different context while they use reading as a form of communication" (NCTM, 1989, p. 27).

Both literature, mathematics and science help us to organize and give order to the world around us. The use of language, in both oral and written forms, and the use of numbers to count, compute, and generate statistical information, provide information that allows us to make decisions daily (Moyer, 2000; NCTM, 2000; Thompson et al., 2008). When language skills are embedded in meaningful contexts, they are easier and more enjoyable for children to learn (Moyer, 2000; NCTM, 2000). In the same way, numbers and operations, when embedded in significant real-world contexts, give children the opportunity to make sense of mathematics and to gain mathematical power (Moyer, 2000; NCTM, 2000).

In addition to the knowledge strands of geometry, patterns, number sense, algebra, and other mathematics skills, NCTM (2000) brought forth the initial emphasis of three process skills that can easily be supported through the use of stories (communication, making connections, and creating representations). Literature certainly introduces a mode of communication for the students because trade books are written with mathematical instruction described in words instead of numbers. The incorporation of literature also provides students to potentially "organize and consolidate their mathematical thinking; develop their mathematical thinking more clearly to their peers and teachers; supports the analysis and evaluation of their mathematical thinking; and utilizes the language of mathematics to express mathematical ideas more precisely" (NCTM, 2000, p.421).

Many children's books present interesting problems and illustrate how other children solve them. For example, in Marilyn Burns' *Spaghetti and Meatballs for All*, 'Mr. and Mrs. Comfort' have to determine various seating arrangements for a family reunion. Initially, there are eight tables carefully arranged with thirty-two chairs so everyone may sit. When guests arrive, they have different ideas for their seating plans. This is a great opportunity for students to explore the varying relationships between area and perimeter using fun familiar situations. Consequently, students are better able to talk about mathematics, because of the exposure to literature (Golden, 2012; Price, 2009). Similarly, when students create representations (i.e., charts, pictures, or patterns) related to the stories, they are not simply math problems on a page, but extensions of the story and characters they just enjoyed (Golden, 2012; Price, 2009).

**Practice/Innovation**

Below, I describe the project designed for the PSTs intended to provide them with opportunities to integrate mathematics [trade] books in their lessons while teaching in their internship classes. I provided them with a sample list of mathematics books provided by *Math Solutions Publications* they might consider, which included appropriate grade levels and content strands (shown in Figure 1). I also presented the PSTs with an example of the NSTA's list of *Outstanding Science Trade Books (OSTB)* for K-12.

**Chart of children's literature featured in the Math Solutions Publications series Math, Literature, and Nonfiction, listed with grade levels and topics.**

| TITLE | AUTHOR | K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | TOPIC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Animal Farm | George Orwell | | | | | | | X | X | X | Percents |
| Annabelle Swift, Kindergartner | Amy Schwartz | | | | | X | X | X | | | Whole Number Computation |
| Anno's Hat Tricks | Akihiro Nozaki | | | | | | | X | X | X | Probability |
| Anno's Magic Seeds | Mitsumasa Anno | | | | | X | X | X | | | Number Sense, Whole Number Computation |
| Bananas | Jacqueline Farmer | | | | X | X | X | | | | Multiplication, Percents |
| Before and After: A Book of Nature Timescapes | Jan Thornhill | X | X | X | | | | | | | Time |
| Benny's Pennies | Pat Brisson | X | X | | | | | | | | Counting, Money |
| Berries, Nuts, and Seeds | Diane L. Burns | | | | X | X | X | | | | Sorting, Graphing |
| Big and Little | Steve Jenkins | X | X | X | | | | | | | Measurement |
| Biggest, Strongest, Fastest | Steve Jenkins | X | X | X | X | X | X | | | | Measurement |

*Figure 1. Screen shot of Children's Literature Chart from Math Solutions Publications*

Throughout the semester the PSTs in my elementary mathematics methods course read several articles that acknowledged the value of incorporating children's [trade] books within content lessons and how they can be used in a variety of ways in the context of learning mathematics and science. A picture book read at the beginning of a lesson may engage students in thinking about their own ideas about a mathematics concept. A nonfiction book may be read to help explain a math concept and encourage inquiry. A fictional story might launch a mathematical investigation. A biography may help a young child develop role models to increase their own self-confidence and interest in mathematics.

I wanted the assignment to be meaningful for the PSTs as well as the students in their classrooms. Initially the PSTs found a STEM literature book and designed a book talk; a concept from NSTA's *Science and Children* journal. There are three principles in which teachers should focus when using trade book integration: (a) engage students to activate prior knowledge; (b) develop competence in an area of inquiry; and (c) recognize that metacognitive approaches to instruction can help students take control of their learning by defining goals and monitoring their progress (NSTA, 2008).

The book talk included a brief synopsis, content and teaching connections, and additional resources they thought were relevant (i.e., lesson plans, activities, and web links associated with the book). In conjunction with completing a book talk, they additionally wrote a brief description of why they selected the particular text, how they intended to use the book (i.e., introduce a topic, connections, or additional support), as well as their own and/or student excerpts discussing key details and any value they found during the experience. Finally, the PSTs included samples of innovative ways in which the inclusion of the literature supported or enhanced their instruction.

## Classroom Examples

### Content and Teaching Connections

One of the PSTs discussed content and teaching connections based on *The Napping House* by Audrey Wood. She read this mathematics book to her kindergarten students hoping to enhance their perception of cardinality and to introduce the concepts of addition and subtraction. Through the use of the read aloud with accountable talk, the students were able to gain an awareness of addition and subtraction processes without realizing they were learning mathematics concepts through the use of questions within the trade books.

Another PST elected to implement *The Cloud Book* by Tomie DePaola in her third grade class during science. The PST selected this book prior to teaching the students about the water cycle to access their prior knowledge and to support their comprehension before participating in a hands-on activity where the students discover a drop of water's journey from ProjectWet (2013). Below she discussed her response to the student reactions during the read aloud:

> My third graders absolutely loved *The Cloud Book*! They laughed hysterically at some of the pictures and words to describe the pictures. They found it neat that the author would talk about what different cultures thought about clouds. Due to the hysterical nature of the book, it kept the kids focused, and wanting me to read on. One little girl

had no clue that snow came from clouds. I asked the kids to share with the class about times they've seen objects in the clouds. They were enthusiastic to share all of their thoughts. I could see this being used as an amazing introductory Science lesson on clouds. This book truly shed a new perspective on clouds, something we see every day. I feel that because there was humor in the book it allowed the children to want to learn more about the names of clouds, and different theories on how clouds were made. Introducing humor into a topic such as clouds showed me that if you put a bit of a silly twist to any subject, children will be eager to learn and further investigate the process of cloud formations and simulation.

The PST also asked the students if they would record their personal reactions to the read aloud on index cards to gain individual awareness and perceptions of the inclusion of literature in content areas (shown in Figure 2).

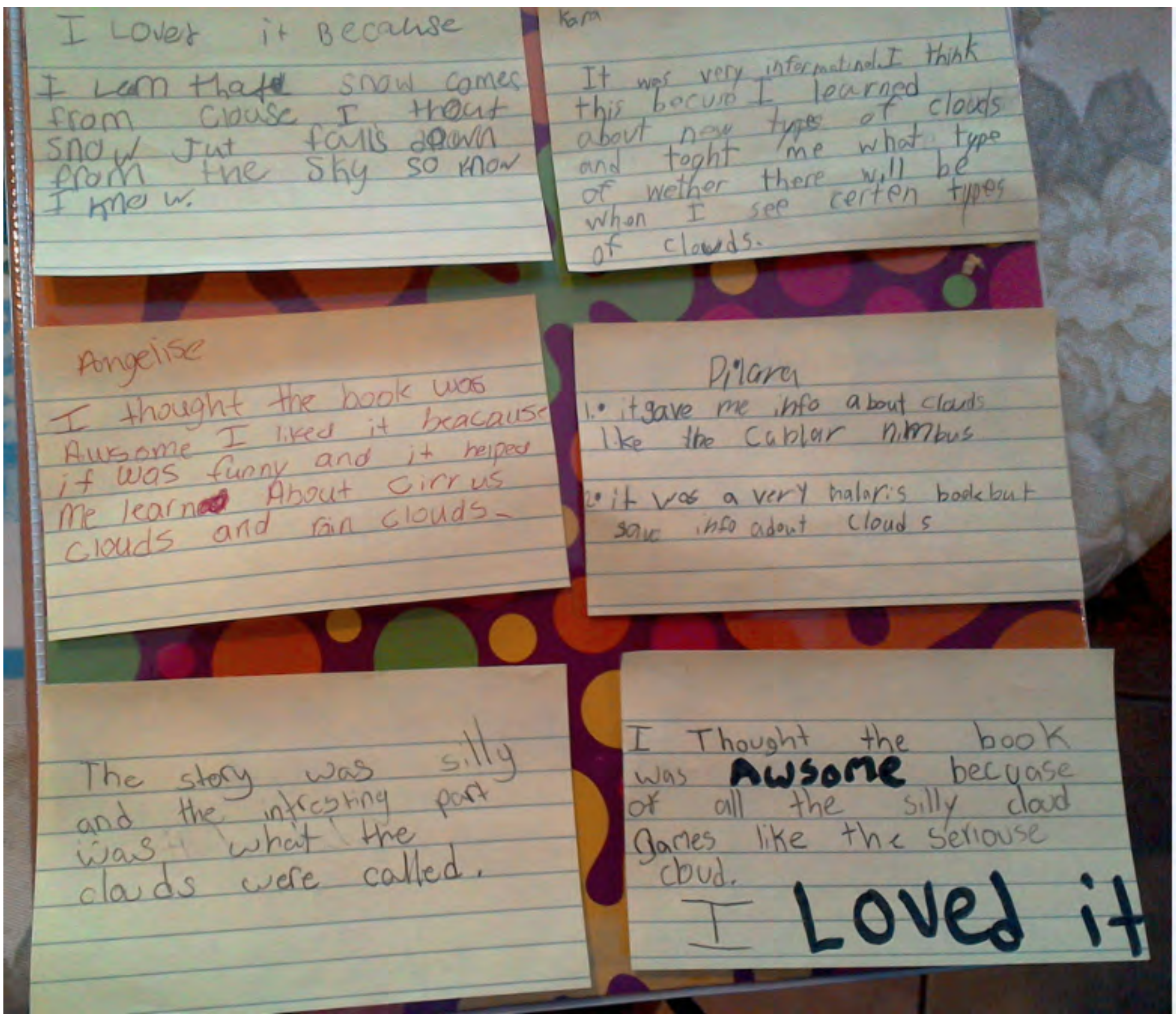



*Figure 2. Screen shot of third grade responses after* The Cloud Book *read aloud*

## Implications

Literacy, particularly reading, is critical across content areas. Mathematics teachers who integrate literature into mathematics recognize that mathematical understanding involves reading and writing (NCTM, 2000). This may also provide students a level of comfort they might typically not have during traditional content lessons (Beane, 1995; Greene, 1991; Hurley, 2001; Rearden & Broemmel, 2008; Tank, 2014). Fittingly, mathematical reasoning and problem solving can be found in authentic reading and writing materials (Moyer, 2000; Ruiz, Thornton, & Cuero, 2010). Proficiency in the area of literacy is critical in the understanding of all content areas. When teachers make a conscious effort to integrate mathematics and science in their daily instruction, this might not only promote students' understandings and appreciation for reading and writing, but also deepens mathematical and scientific conceptualizations (Draper, 2002; Pearson et al., 2010; Thompson et al., 2008).

Understandably, not every book will be appropriate for enhancing a mathematics or science lesson and there should be a natural relationship between the book chosen to enhance the lesson and lesson itself (Thompson et al., 2008). It is also critical to note that incorporating informational and story books in the curriculum in the early years of school has the potential to increase student motivation, build important comprehension skills, and lay the groundwork for students to grow into confident, purposeful readers (Golden, 2012; Price, 2009). Researchers and educators have developed criteria for selecting literature that can be integrated effectively within content instruction. (Worley, 2002; Price, 2009; Rearden & Broemmel, 2008).

Being aware of quality trade books and their usefulness in mathematics and science teaching is a necessary first step in successfully teaching through literature and subsequently may help strengthen NCTM and the NRC's advocacy for high quality mathematics teacher education in support of quality mathematics teaching.

## References


DePaola, T. (1975). *The Cloud Book*. New York: Holiday House.

Douville, P., Pugalee, D. K., & Wallace, J. D. (2003). Examining instructional practices of elementary science teachers for mathematics and literacy integration. *School Science and Mathematics, 103*, 388-396.

Golden, L. (2012). Children's literature in mathematics instruction. *Library Media Connection, 31*(2), 40-42.

Miners, Z., & Pascopella, A. (2007). The new literacies. *District Administration, 43*(10), 26-34.

Moyer, P.S. (2000). Communicating mathematically: Children's literature as a natural connection. *The Reading Teacher, 54*, 246-256.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2014). *Principles to Actions: Examining Mathematical Success for All.* Reston, VA: NCTM.

National Research Council. (2000). *National science education standards*. Washington, DC: National Academy Press.

National Science Teacher Association. (2008). *Using science notebooks in elementary classrooms.* Retrieved from http://static.nsta.org/files/PB209Xweb.pdf

Pearson, D. P., Moje, E., & Greenleaf, C. (2010). Literacy and science: Each in the service of the other. *Science, 328*, 459-463.

Price, R., & Lennon, C. (2009). *Using children's literature to teach mathematics: The Quantile framework for mathematics.* Available from http://quantiles.com/resources/LiteratureMathematics.pdf.

Rearden, K. T., & Broemmel, A. D. (2008). Beyond the talking groundhogs: Trends in science trade books. *Journal of Elementary Science Education, 20*(2), 39-49.

Ruiz, E., Thornton, J., & Cuero, K. (2010). Integrating literature in mathematics: A teaching technique for mathematics teachers*. School Science and Mathematics, 110*(5), 235-238.

Thompson, D. R., Kersaint, G., Richards, J. C., Hunsader, P. D., & Runenstein, R. N. (2008). *Mathematical literacy: Helping students make meaning in the middle grades.* Portsmouth, NH: Heinemann.

Ward, R. A. (2009). *Literature-based activities for integrating mathematics with other content areas.* Boston, MA: Pearson.

Wood, A. (1984). *The Napping House.* San Diego: Harcourt Brace Jovanovich.

# USING MATH NOTEBOOKS FOR MATHEMATICAL INVESTIGATIONS: ENGAGING PROSPECTIVE TEACHERS IN "DOING" MATHEMATICS

Michelle T. Chamberlin
University of Wyoming
mchambe5@uwyo.edu

*Too often, students are merely consumers of mathematics, without many opportunities to engage in authentic processes of doing mathematics. Math notebooks, in which students document their processes while working on mathematical problems, serve as one means to address this dilemma. I describe the use of math notebooks with 'low-threshold, high-ceiling' mathematical investigations in my undergraduate mathematics class for elementary education majors. Details about the implementation and impacts are shared, including how the notebooks extended the prospective teachers' perceptions of doing mathematics.*

## Introduction

To support their learning, students need authentic experiences in doing mathematics, including opportunities to read and write mathematics as mathematicians do (e.g., Schmoker, 2011). As a mathematics teacher educator, I decided to ask prospective teachers (PSTs) to produce mathematical texts reflective of doing mathematics. Specifically, I asked PSTs in my undergraduate mathematics class for elementary education majors to use math notebooks while working on 'low-threshold, high-ceiling' mathematical investigations. Low-threshold, high-ceiling mathematical tasks allow students to begin at their own levels of engagement while simultaneously providing space for advancing to higher level mathematics (McClure, 2011). Math notebooks document one's thinking and processes while working on such mathematical investigations. My guiding questions for the classroom investigation were: What are the PSTs' perceptions of doing mathematics after completing the math notebooks and the mathematical investigations? How do these compare to their prior views? The purpose of this paper is to describe the implementation of the math notebooks and investigations along with their associated impacts for others considering adopting a similar approach in their own grade 5-16 mathematics or science classrooms.

## Instructional Framework and Related Literature

The end-products of mathematical work, including journal articles, textbooks, and lectures, make heavy use of formalized symbolic systems, proceed deductively from axioms to conclusions, and are impersonal and dense in content (Morgan, 1998). Such presentations leave most individuals with a perception of mathematics as algorithmic, step-by-step, and only consisting of deductive logic (Byers, 2007). Yet, the authentic work of mathematicians includes much more, such as informal processes, imagination, intuition, and creativity (Burton, 1999;

Henrion, 1997). Doing mathematics is not without struggle or challenge (Burton, 2004; Byers, 2007). Upon 'being stuck', mathematicians may take a break, discuss the difficulty with someone else, read journals for further information, utilize technology, or work on another problem. Mathematicians do not regard 'being stuck' as a negative occurrence, rather as a potential for further learning (Henrion, 1997).

This picture of doing mathematics is often absent from mathematics classrooms (Burton, 2004; Henrion, 1997). When only presented with mathematical reasoning in polished form, students experience mathematics as someone else's discipline for which they are merely expected to memorize and reproduce results as "passive consumers" (Schoenfeld, 1988, p. 160). As a result, students demonstrate counterproductive conceptualizations of mathematics (Schoenfeld, 1988):

1. Formal mathematics has little to do with invention or discovery.
2. Students who understand mathematics can solve mathematics problems very quickly.
3. Only very intelligent individuals are capable of understanding or creating mathematics.

In response, many educators are advocating that students engage in experiences reflecting the work of mathematicians, e.g., as *doers* of mathematics (Burton, 2004; Davis, Hersh, & Marchisotto, 2012; Morgan, 1998). One avenue is mathematical writing or "writing-to-learn" (Connolly, 1989).

Writing-to-learn is not concerned with demonstrating mastery or regurgitating information, but rather with helping students acquire conceptual understandings. It has been shown to enhance student learning, achievement, and engagement in multiple subjects across various grade levels (e.g., Bangert-Drowns, Hurley, & Wilkinson, 2004; Reynolds, Thaiss, Katkin, & Thompson, 2012; Rivard, 1994). However, contextual factors, including the type of writing, influence the relationship between writing and learning (Bangert-Drowns, Hurley, & Wilkinson). Thus, many educators draw upon two classifications: transactional writing and expressive writing (Britton, Burgess, Martin, McLeod, & Rosen, 1975). Transactional writing is used to inform, persuade, or instruct an external audience through clear, conventional, and concise prose focused on a final product. Common examples include term papers, book reviews, journal articles, and laboratory reports. Expressive writing externalizes and clarifies a writer's own thinking through informal, exploratory writing, often described as thinking aloud on paper. It may include feelings as well as thoughts about a problem, issue, or text and is typically found in journals, letters to close friends, and freewriting. Transactional writing has been found to constitute a substantial percentage of student writing in middle through post-secondary classrooms (Applebee, 1984; Britton et al., 1975; Melzer, 2009). This overemphasis on transactional writing reduces students' opportunities to engage in creative, sense-making, critical, and independent thinking processes (Fulwiler, 1982). In contrast, Fulwiler (1982) and King (1982) recommend students begin with expressive writing and transition to transactional writing. As such, expressive writing is a means for enhancing transactional writing.

## Innovation: Mathematics Notebooks for Mathematical Investigations

The mathematics class was a number and operations course with 33 elementary education majors, 29 of whom consented to participate in the research. I provided the PSTs with 10 low-threshold, high-ceiling mathematical investigations from which they selected two to work on in their Mathematics Notebooks. A sample mathematical investigation is provided below:

> **Fred's Baseball Cards:** Fred takes his collection of baseball cards and places them on the floor in piles of two. He has one card left over at the end. He gathers the cards together and then lays them out in piles of three. Again, he has one left over. He tries one more time, this time laying the cards out in piles of four; once again he has one card left over. Exasperated, Fred gathers up his cards, lays them out in piles of seven, and this time he has no cards left over. How many cards does Fred have in his baseball card collection? Are there other solutions? What are they? Are there any patterns that will help you generate solutions? Consider the following similar problem which appeared in a Chinese text written over 17 centuries ago: For a number that is unknown, when divided by 3, the remainder is 2; when divided by 5, the remainder is 3, and when divided by 7, the remainder is 2. What is the number? Are there other solutions? What are they? Are there any patterns that will help you generate solutions? How is this similar to, and different from, Fred's Baseball Cards? (Adapted from Romagnano, 2003).

To introduce the mathematics notebooks and investigations, I explained the investigations were substantial in nature so the mathematics notebooks would enable the PSTs to record and coordinate their thinking across multiple sessions. Next, I described the Notebook Guidelines, shared an example mathematics notebook for a different mathematical investigation, and handed out the 10 investigations.

Notebook Guidelines:

1. Include your calculations, computations, diagrams, graphs, pictures, etc. **PLUS** your thinking behind such work, e.g., your questions, plans, suspicions, findings, ah-ha moments, mathematical decisions, helpful or not helpful steps or resources, etc.
2. Write enough that you will be able to remember what you did four months from now.
3. Feel welcome to tape in data or diagrams generated by technology.
4. Work in pen, and no ripping out pages, whiting out, or scribbling through work.

Throughout the semester, the PSTs worked individually on their two mathematical investigations as part of various homework assignments. For each investigation, the PSTs turned in a 4-10 page Mathematics Investigation Write-Up. Before each write-up, the PSTs had 5-10 minutes in class to discuss their mathematical investigations, which they were welcome to continue outside of class as desired.

Mathematics Investigation Write-Up

*Part I Mathematical Work:* For at least 3 sub-questions, describe your mathematical findings AND provide explanations and justifications. *Part II Personal Experience:* Describe the path that you took in working on this mathematical investigation, e.g.,

- Strategies that helped you work on the mathematics and why they were helpful;
- "Ah ha!" moments;
- Times when you felt stuck and how you got going again;
- Surprises and how they affected your thinking about the mathematics;
- Mathematical issues or challenges you faced and how you addressed them;
- Errors or misconceptions you had, how you fixed them, and how they impacted your work;
- Resources that you drew upon and how; and/or
- Your emotional reactions and when – frustration, excitement, curiosity, determination, etc.

I graded and provided feedback on each write-up. The PSTs then selected one of their mathematical investigations to revise for their Polished Math Investigation Write-Up. Finally, students completed a 3-10 page Reflection on Math Investigations.

Reflection on Math Investigations

*Part I Preparation:* Make notes about the following in your math notebook: 1. How did the Math Notebook and the Math Investigations impact your view of mathematics and your perceptions of how to make sense of and understand mathematics? 2. Review your Depiction of a Student "Good" at Math. How have the Math Investigations and/or our class impacted your view of a student "good" at math? 3. Read the article *I'm Not a Math Person Is No Longer a Valid Excuse* (Dickerson, 2013). How has this reading further informed your view of mathematics and your perceptions on how to make sense of and understand mathematics? *Part II Writing Your Paper:* Describe your current views on mathematics AND on how to make sense of mathematics. How have your views changed since the beginning of the semester? What caused those changes? How might your new view impact your future teaching of mathematics?

My analyses to date includes classifying as Advanced, Proficient, or Developing each PST's math notebook in terms of documenting their mathematical thinking and each PST's first Math Investigation Write-Up in terms of the depth of their reflection. I have also used open coding (Strauss & Corbin, 1998) on the first Math Investigation Write-Ups and the Reflections on Math Investigations to identify themes about the PSTs' views on doing mathematics.

## Classroom Results and Examples

With regard to documenting their mathematical thinking in their math notebooks, 44% (12) of the PSTs' math notebooks were Advanced, 48% (13) Proficient, and 8% (2) Developing. Below are brief examples from an Advanced and a Developing notebook. In terms of the depth of the PSTs' reflections in the first Math Investigation Write-Ups, 57% (16) were Advanced, 32% (9) Proficient, and 11% (3) Developing. Thus, for nearly all PSTs, both their notebooks and write-ups provided rich material for reflection on their views of doing mathematics.

Seventy-nine percent of the PSTs reported positive experiences with the math notebooks and investigations, including 17 PSTs (58%) that acknowledged negative experiences or perspectives of mathematics in previous classes. These positive experiences included feelings of accomplishment and excitement, often associated with ah-ha moments. The PSTs also reported several significant realizations about mathematics.

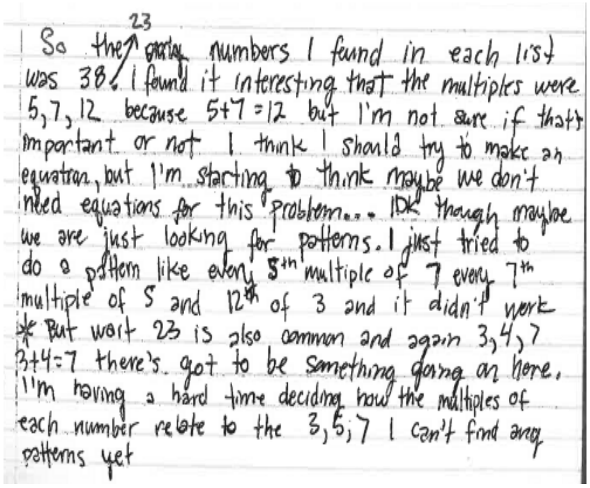

So the 23 numbers I found in each list
was 38. I found it interesting that the multiples were
5, 7, 12 because 5+7=12 but I'm not sure if that's
important or not. I think I should try to make an
equation, but I'm starting to think maybe we don't
need equations for this problem... IDK though maybe
we are just looking for patterns. I just tried to
do a pattern like every 5th multiple of 7 every 7th
multiple of 5 and 12th of 3 and it didn't work
* But wait 23 is also common and again 3, 4, 7
3+4=7 there's got to be something going on here.
I'm having a hard time deciding how the multiples of
each number relate to the 3, 5, 7 I can't find any
patterns yet

*Figure 1*. *Advanced example of commentary.*

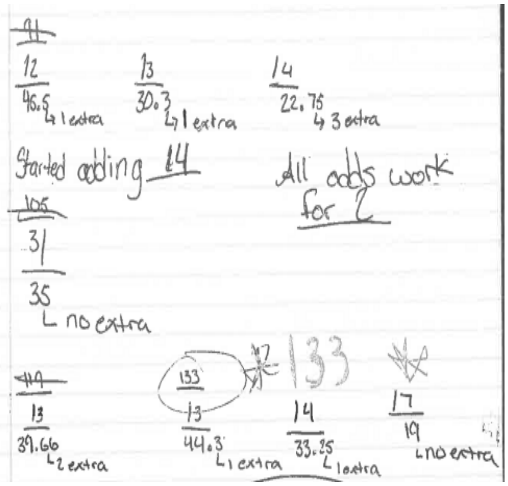



*Figure 2*. *Developing example of commentary.*

The most common theme was recognizing that mathematical problems may be approached and solved in many different ways (83%):

> Previously my views on mathematics led me to believe that you can only approach a topic in one particular way … I now make sense of mathematics by knowing that all students have a different thinking about starting the problems … An example of this is when I worked on the Fred's Baseball Cards problem. I collaborated with another student … once we started looking at the patterns we found, they were different because I had started with my answer of 49, where she started with 7, which was a multiple of the answer.

Next, 58% of the PSTs reported realizing that it is important to understand 'why' in mathematics:

> Before taking this math class I thought of math as just memorizing equations and not really learning concepts for future knowledge. My second math investigation led me to the new understandings of why certain equations work the way they do. … After understanding how and why the problem worked the way it did, my views about equations just being some magical way to solve problems had changed.

Another significant insight by the PSTs was the realization that mathematics is not always a straightforward process. At least 13 (45%) acknowledged that one does not always know what to do next or always do everything correctly in working on a mathematical investigation. Furthermore, 8 PSTs (28%) recognized that such mis-steps, while frustrating, often lead to further insights:

> I also found that when I did something that was not necessarily correct or did not necessarily lead to answers often helped me. Sometimes those 'mistakes' led me to a new strategy that was actually really helpful. …Mistakes are a natural part of learning and they often help. Mistakes do not mean failure, mistakes mean growth.

Many (58%) PSTs reported such cycles of highs and lows, sharing their negative moments but also explaining that they overcame such frustrations. Two other prominent realizations by the PSTs included recognizing that the mathematical process is more important than determining the "right answer" (41%) and that within a mathematical investigation one insight often leads to another (24%).

Much like mathematicians, the PSTs noted several helpful approaches for dealing with mathematical challenges, including taking breaks, having patience, creating visuals, collaborating with others, viewing the problem from a different angle, acting out the problem, and listing all possibilities or cases. One PST explained the value of collaborating:

> So I came back to the problem a few days later and I was still struggling … I then decided that maybe I should try to work with some other people. So I met up with some girls on my floor and we talked it out. It was cool because we each had found something the others did not see. We shared our input and we helped each other with things when they were stuck. Sometimes when you explain your work or try to help someone you understand the problem so much better for yourself.

Finally, many of the PSTs enhanced their perception of mathematical ability. After the investigations, the PSTs reported being 'good' at math is about effort (e.g., taking time, practice, hard work) (72%) and a positive disposition (e.g., motivation, persistence, desire) (62%). Previously, the PSTs perceived that individuals either 'got' math or they 'didn't' and that students 'good' at math memorize formulas and theorems, always have all the answers, do not have to work hard at understanding mathematics, work alone, and do not need help or assistance.

## Discussion

As a result of completing the mathematics notebooks and investigations, the PSTs enhanced their perceptions of the authentic work of doing mathematics. They recognized that mathematical problems may be approached in more than one way, that it is important to "understand" mathematical concepts, and that doing mathematics is not always a straightforward process. The PSTs also articulated many strategies for facing mathematical challenges and realized that mathematical achievement is not innate but instead a result of effort and disposition. Furthermore, many PSTs mentioned how the math notebooks and investigations may influence their future teaching. Realizing that mathematical problems may be completed in different ways led many to plan to encourage elementary students to invent their own strategies (31%), to use multiple methods in their teaching of mathematics (28%), and to make sense of elementary students' thinking because their thinking may appear different but be logical (31%). In addition, six PSTs commented that they plan to incorporate problem solving in their future elementary classrooms. With regard to writing-to-learn, the math notebooks and investigations demonstrated the potential of asking students to begin with expressive writing (the math notebooks) and gradually transition to transactional writing (the math investigation write-ups and reflections). In comparison to writing assignments completed by PSTs in previous sections of this class, I found the PSTs' mathematical write-ups to be more thorough, accurate, and explanatory and their reflection papers to be deeper and more detailed.

## Implications

Asking students to utilize math notebooks while working on mathematical investigations has the potential to engage students as "doers" of mathematics. In addition, engaging students in expressive to transactional writing may improve their mathematical understandings. Such activities may be used with mathematics or science students in grades 5-16. Indeed, many ideas for the math notebooks arose from similar efforts with science notebooks (Ruiz-Primo, Li, Ayala, & Shavelson, 2004). In looking forward, I hope to decrease the significant time involved in grading the two write-ups, polished write-ups, and reflections while still maintaining the essence of the math notebooks and investigations. Some possibilities include using peer reviews, having PSTs work on two investigations but turn in one write-up, or dropping the polished write-up. Second, the number and operations course is the first in a three-course sequence for our elementary education majors. What might be done in the second and third courses? More mathematical investigations? Could the math notebooks and investigations be a springboard to address pedagogical topics such as low-threshold, high ceiling tasks and writing-to-learn? For research, I intend to finish coding the second Math Investigation Write-ups and the Polished Math Investigations to finalize the themes identified here. Second, I plan to use the *Multi-Dimensional Problem-Solving Framework* of Carlson and Bloom (2005) to analyze the PSTs' cyclic nature of problem solving on the mathematical investigations. Completing this analysis will provide a more thorough examination of the PSTs' mathematical work and problem solving on the mathematical investigations, supplementing the PST self-report data here.

## References


Applebee, A. N. (1984). *Contexts for learning to write*. Norwood, NJ: Ablex.

Bangert-Drowns, R. L., Hurley, M. M., & Wilkinson, B. (2004). The effects of school-based writing to learn interventions on academic achievement: A meta-analysis. *Review of Education Research, 74*(1), 29-58.

Britton, J., Burgess, T., Martin, N., McLeod, A., & Rosen, H. (1975). *The development of writing abilities* (11-18). *Schools Council Research Studies Series.* London, Macmillan.

Burton, L. (1999). Why is intuition so important to mathematicians but missing from mathematics education? *For the Learning of Mathematics, 19*(3), 27-32.

Burton, L. (2004). Mathematicians as enquirers: Learning about learning mathematics. In A. J. Bishop (Ed.), *Mathematics education library* (Vol. 34). Boston, MA: Kluwer.

Byers, W. (2007). *How mathematicians think: Using ambiguity, contradiction, and paradox to create mathematics*. Princeton, NJ: Princeton University Press.

Carlson, M. P., & Bloom, I. (2005). The cyclic nature of problem solving: An emergent multidimensional problem-solving framework. *Educational Studies in Mathematics, 58*, 45-75.

Connolly, P. (1989). Writing and the ecology of learning. In P. Connolly & T. Vilardi (Eds.), *Writing to learn mathematics and science* (pp. 1-14). New York: Teachers College Press.

Davis, P. J., Hersh, R., & Marchisotto, E. A. (2012). *The Mathematical Experience, Study Edition*. Boston, MA: Birkhauser.

Dickerson, E. (2013, November). Being good at math is not about natural ability. *Business Insider*. Retrieved from http://www.businessinsider.com/being-good-at-math-is-not-about-natural-ability-2013-11

Fulwiler, T. (1982). *Writing: An act of cognition*. In C. W. Griffin (Ed.), *Teaching Writing in All Disciplines* (pp. 15-26). San Francisco, CA: Jossey-Bass.

Henrion, C. (1997). *Women in mathematics: The addition of difference*. Bloomington, IN: Indiana University Press.

King, B. (1982). *Using writing in the mathematics class: Theory and practice*. In C. W. Griffin (Ed.), *Teaching writing in all disciplines* (pp. 15-26). San Francisco, CA: Jossey-Bass.

Melzer, D. (2009). Writing assignments across the curriculum: A national study of college writing. *College Composition and Communication, 61*(2), 240-261.

Morgan, C. (1998). Writing mathematically: The discourse of investigation. In P. Ernest (Series Ed.), *Studies in Mathematics Education Series: Vol.* 9. Bristol, PA: Falmer Press, Taylor & Francis, Inc.

McClure, L. (2011, September). *Using low threshold high ceiling tasks in ordinary classrooms*. Retrieved from http://nrich.maths.org/7701

Reynolds, J. A., Thaiss, C., Katkin, W., & Thompson, R. J. (2012). Writing-to-learn in undergraduate science education: A community-based, conceptually driven approach. *Life Sciences Education, 11*, 17-25.

Rivard, L. P. (1994). A review of writing to learn in science: Implications for practice and research. *Journal of Research in Science Teaching, 31*, 969-983.

Romagnano, L. (2003). *Mathematics for elementary teaching: An instructor's guide*. Department of Mathematical and Computer Sciences, The Metropolitan State College of Denver.

Ruiz-Primo, M. A., Li, M., Ayala, C., & Shavelson, R. J. (2004). Evaluating students' science notebooks as an assessment tool. *International Journal of Science Education, 26*(12), 1477-1506.

Schmoker, M. (2011). *Focus: Elevating the essentials to radically improve student learning*. Alexandria, VA: ASCD.

Schoenfeld, A. H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist, 23*(2), 145-166.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research* (2nd ed.). Thousand Oaks, CA: SAGE.

## Acknowledgements

Many thanks to Jodie Novak and Frieda Parker from the University of Northern Colorado for introducing me to the use and potential of Math Notebooks.

# UTILIZING CULTURALLY RELEVANT STORIES IN MATHEMATICS

Amy Corp
Texas A&M Commerce
Amy.Corp@tamuc.edu

*Classrooms with increasingly diverse populations demand innovative pedagogies that engage all students. This paper briefly explores the research conducted in recent years in reaching African American students in ways that affirm their culture. One such practice capitalizes on stories that feature African Americans to increase engagement during language arts and social studies classes. The results of transferring this pedagogy to the mathematics classroom are explored in this study. An example from recent practice, as well as, vetted stories that align to specific mathematical topics in elementary curriculum give educators resources for affirming and engaging all students.*

## Introduction

Mathematics as it is currently being taught and assessed has proven to be difficult for students, particularly students of color. To subvert this trend, innovative ideas for culturally responsive teaching in mathematics must be explored. Culturally responsive teaching means that educators connect to students' home and cultural funds of knowledge (Gay, 2002). Educators must also affirm how mathematics is part of their world with messages of a growth mindset (Boaler, 2016). A growth mindset in mathematics is the belief that one can learn math, that we are not born smart or dumb in math. One way to meet both of these objectives is to utilize stories that feature the culture of marginalized students and then use the context of that story to provide opportunities to learn and do mathematics (review operations, problem-solve, introduce concepts, practice reasoning, etc.). This is in direct contrast to some traditional practices of computation drills, modeling, or memorizing steps to solve problems in isolation.

## Purpose of the Paper

The purpose of this article is to demonstrate how to engage African American students in mathematical tasks that promote a growth mindset by utilizing culturally responsive stories. Not all stories featuring African American characters are culturally responsive because they are not all affirming to that culture. This research utilized stories that have been vetted using standards designing by Heflin and Barksdale-Ladd (2001) for stories qualifying as affirmative to the African American culture. This author hopes to motivate educators to seek such stories for cultures mirrored in their own classrooms, to use them with mathematical tasks that will foster a growth mindset, and to promote this practice as progress toward equity in mathematics.

## Significance, Related Literature, and Theoretical Framework

This research is significant because mathematics has traditionally been a difficult subject for students of color, especially when compared to the dominant population (reflected in state scores, media, and a high volume of students of color in intervention or retention classes). Little has changed to engage these students, despite an increase in aggressive drill and practice tactics further indicating that teaching practices need to change, especially for students of color (Blank, 2011; Delpit, 2012).

Research in cultural responsive teaching demonstrates that students of all cultures respond to stories and examples which reflect their own culture (Gay, 2002; Heflin, 2002; Ladson-Billings, 1995, 1997). This has been quite successful in teaching language arts (Tatum, 2006) and in communicating social studies themes (Walker-Dalhouse, 1992) but has not been researched in mathematics. In fact, research into the textbooks of elementary mathematics reveals little positive connection to people of color in examples or photos (Sleeter & Grant, 2011).

Yet research into mathematics pedagogy reveals that story is a powerful medium to engage students in thinking about mathematics (Moyer, 2000; Schiro, 1997; Shatzer, 2008). How so? Stories transport students into the action of the story by holding their attention with picturesque wording and vivid visuals. When the story has characters doing mathematics, (baking, shopping, measuring, counting, sorting, collecting data, problem solving, etc.) students are inspired to think about it as if they were in the story. This thinking about the mathematics in a realistic situation becomes powerfully engaging. Stories are encouraged for use in introducing a concept, reviewing a concept, problem-solving, reasoning, and making connections to the real world (Hong, 1995; Whitin & Whitin, 2004). Yet at this time, no published research demonstrates how culturally responsive stories (those featuring non-White characters) are utilized in mathematics.

This framework combines the strategy of culturally responsive stories with the use of stories in mathematics. By implementing this framework the research adds to our capacity for multicultural education in a subject often void of connections to students' home life or culture. A story that reflects marginalized students affirms that student population, and culture to themselves and the class (Heflin, 2002; Ladson-Billings, 1997). Stories also give all students the same background information when the connecting lesson or tasks are based on the story creating a more equitable playing field (Perez, 2012; Moyer, 2000).

By scrutinizing dozens of stories featuring African American as culturally responsive, and then defining the mathematical connections that can be made to these stories, Teachers are provided with practical ways to use culturally responsive stories in mathematics. An example using results from the first story in this research is included in the classroom example section.

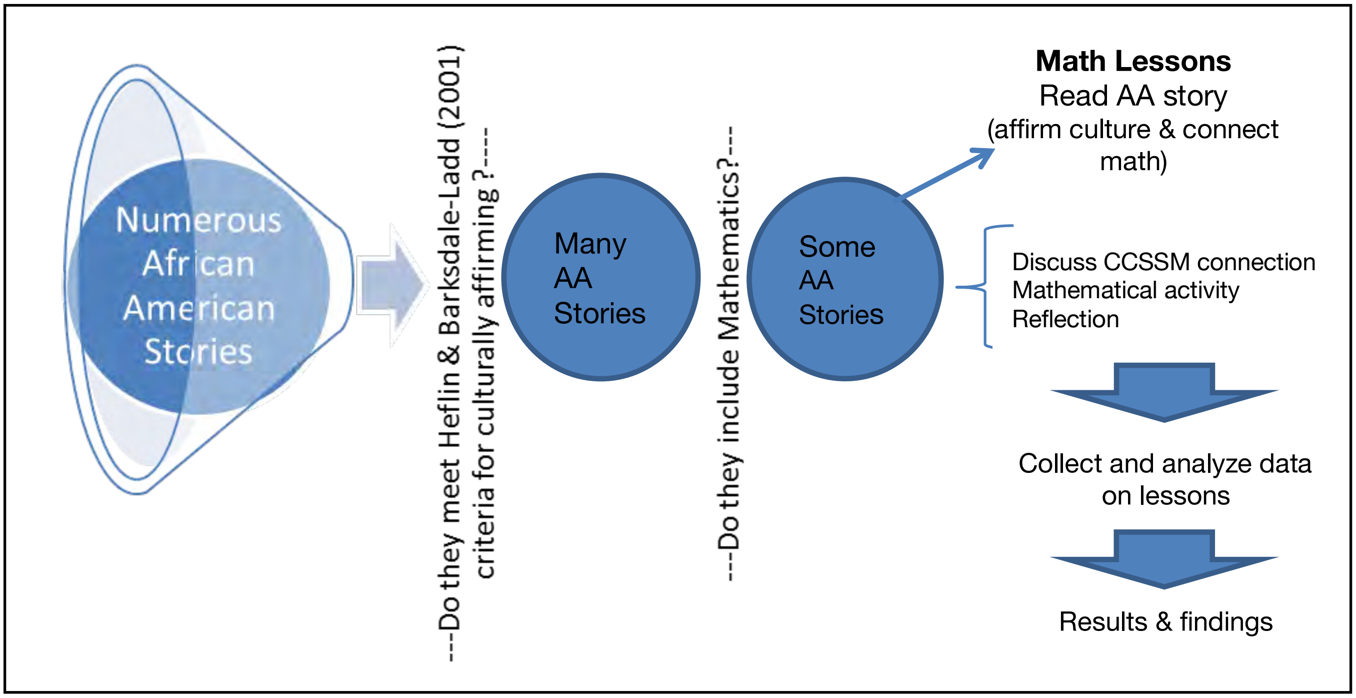



*Figure 1. Framework for choosing and implementing stories featuring African American culture.*

## Innovation

So how do African American children respond to culturally relevant stories in mathematics? To explore this research question, two third grade mathematics classes were led by their teacher in a story and corresponding activity once a week for eleven weeks. The race of the 41 participating students were identified to their public school (by parent) as: 17 Black (African American), 17 White (Caucasian) and seven Hispanic (Latino/a).

The lessons were designed with the culturally responsive story as the focal point for the connecting discussion, task (or activity), and individual reflection. Each story was carefully selected to meet the criteria for being culturally responsive to the African American culture and to align with the mathematical standards in the school's curriculum for that semester. (See table below for a sample list of stories and their mathematics focus.) As students listened to the story, discussed the connections between the story and the mathematical concept of the

*Table 1.*

Sample of culturally responsive stories for use in mathematics

| Story Title and author | Mathematical focus and possible task |
|---|---|
| *Wilma unlimited: How Wilma Rudolph became the world's fastest woman.* (Krull, & Diaz, 2000). | Measurement (distance, time), problem solving, graphing, plotting, create own data by running her races |
| *Salt in His Shoes: Michael Jordan in pursuit of a dream* (Jordan & Jordan, 2000) | measurement, operations with problem solving, ratios, create data with 2 and 3 point shots by group, graphing |
| *Just Like Josh Gibson* (Johnson & Peck, 2007) | distance, measuring, collecting data (they find their batting average), graphing, predicting, problem posing and solving |
| *Sweet Potato Pie* (Lindsey, 2003) | measurements (capacity), equivalent fractions, problem solving; students create plan for selling pies to save the farm |
| *Just Right Stew* (English, 1998) | measurements, ratios, students recreate recipe for more family (or the class), or with limited measuring tools |
| *Gettin' Through Thursday* (Cooper, 2000) | financial literacy, students plan celebration gift (or party) on a budget |
| *Auntee Edna* (Smothers, 2001) | capacity, measuring, elapsed time, arrays, problem solving and posing |
| *Lucky Beans* (Birtha, 2010) | estimation, measurement, design plan to feed more family (or the class), create plan and estimate real jar of beans |
| *Henry's Freedom Box* (Levine, 2016) | distance/time, volume of crate, proportions, ratios, problem solving, probability, create crate for own travel |
| *Stichin' and Pullin' A Gee Bend Quilt story* (McKissack – 2008) | area, perimeter, shapes, patterns, designing their own quilt, problem solving with limited materials |
| *Ruth and Green Book* (Ramsey, 2010) | distances, elapsed time, reasoning, planning, problem-solving students determine a possible route with parameters from the story |
| *Queen of the Track* (Lang, 2012) | distance, time, problem-solving, creating their own data by jumping (charts, graphing) |
| *Grandma's Gift* (Velasquez, 2010) | time, purchasing (by portion), weight, cooking, shapes (nets, surface area), design on gift with parameters |
| *She Loved Baseball: The Effa Manley Story* (Vernick, 2010) | time, income, (schedule, ordering, purchasing, arranging transportation) problem solving, students plan their own team schedule in small groups |

week, and practiced the mathematics during the task related to the story, the author/researcher took observation notes.

The students completed a short reflection (of three circle-a-response type question) after each lesson. These were analyzed along with student work and the observational notes for each lesson, through constant comparison analysis (Glaser, 1965) for patterns and through simple quantitative analysis for positive and negative reactions to the story (for example; frequency counts on student story ratings and student interaction, averages of interactions and responses during story, ratio of responses to number of students).

**Classroom Example**

Results from students' reflections showed extremely positive reactions to the stories from all students, with the most positive reaction coming from African American students. 100% of these students rated the stories as *interesting* or *awesome* on their reflection. Observation notes of their interaction with the story (mimicking the actions, reacting verbally, agreeing with characters and clapping at the end) concur that these students, were particularly engaged with the story.

For example, the first story about Wilma Rudolph (Krull & Diaz, 2000), was rated as *awesome* by 16 of the 17 African American students (the other rated it as *interesting*) on their reflection. Observation notes reveal that many of them responded verbally during the story, most reacting to her family life, or the injustice of not being treated at the hospital nor being allowed to play basketball. Here are some examples: "Was she the youngest, like me?", "That's not fair!", "That was before Dr. King." All students clapped at her triumphant ending. Some chose to verbalize as well; "Ah, she can run!", "Man, she's good."

Students also responded positively about the impact of the story in helping them to think about mathematics. African American students indicated this positive impact in their circled responses of "it helped a lot", or "it helped some" on their reflection sheet. None of the students chose the negative response ("it did not") to this question for any of the stories.

Results from observation notes triangulate their positive reaction. For example, during the task (collecting data from their own running on a 200 meter section marked by cones every ten meters) students demonstrated strong engagement. Every student participated willingly while meeting the CCSS.Math.Content.3.NBT.A.1which involves rounding to the nearest 10, as well as CCSS.Math.Content.3.NBT.A.2, which involves using strategies to add and subtract up to 1000, as they compared their own distance in 10 and 30 second intervals to Wilma's. Students ran five at a time and stopped at the whistle for 10 second interval, and then raced again for a 30 second interval. Their partner helped them identify which cone they were closest to so the runner could experience the principle of rounding. They recorded their own distances and compared them to Wilma's estimated distance at these interval based on her gold medal performance in the 1960 Olympics in Rome (https://www.olympic.org/wilma-rudolph).

*Figure 2. Example of question correlated to the story in week one.*

Wilma ran the 100m dash is 11.0 seconds. That's 9.09 meters per second! When we round we use 9 meters per second to see that she ran just over 90 meters in 10 seconds. She ran the 200m dash in 24 seconds this is about 240 in 30 seconds if she continued at that speed. Compare your distance. How much farther could Wilma run in the same time?

| Name | Distance in 10 seconds | Distance 30 seconds |
|---|---|---|
| Wilma Rudolph | about 90 meters | about 240 meters |
| YOU: | | |

WOW!

Wilma went ___________meters farther in 10 seconds than I did.

Wilma went _________meters farther in 30 seconds than I did. She was **FAST**!

https://www.olympic.org/wilma-rudolph

Students seemed eager to work in groups to determine their average distances and to input this data into a class graph for comparison to Wilma's. Their written work also displayed engagement with all spaces completed, computations on the side and even some exclamations and doodles. Many students asked if they could run a second time to see if they could come closer to her (Wilma Rudolph's) time. These traits of enthusiasm, quickness and eagerness to complete the work all contribute to a growth mindset with the message that students learn mathematics from doing mathematics.

Throughout the study similar results occurred for each story, some more exciting than others, but all engaging and earning positive reactions from all students, in particular African Americans.

## Implications

So what does this mean for best practices and teaching diverse classrooms? This study demonstrates that all students reacted positively to the use of African American stories even if that was not their culture. All students of color (and nearly all Caucasian students) responded that the stories helped them think about mathematics. Being engaged in the thinking of mathematic is crucial to learning and doing mathematics. Our current best practices in mathematics are based on growth mindsets and these occur only when students are thinking, reasoning and connecting (Boaler, 2016). Using tasks connected to stories all students had just heard proved to be a powerful means for connection. Students seemed to identify with the character in the story and when the task involved solving similar problems to that of the character, they were engaged and ready to think about how to solve the mathematical task. African American students in particular seemed confident to try the task connected to the stories.

Do these positive reactions come only from African American stories? I believe the practice of using culturally responsive teaching with other cultures should prove just as engaging and positive because students can see themselves in the story while a non-dominant culture is affirmed to the class, giving everyone confidence to join the task that corresponds to the story. My work has expanded to explore stories featuring Hispanic characters in mathematics and so far promises to have positive reactions from all students.

Using stories is already a strong research-based mathematical practice to engage students in thinking. Purposefully choosing stories that mirror our marginalized students empowers the story to engage more learners and affirms diversity within the classroom. This practice should foster or start growth mindsets for students of color to really do and learn mathematics, as they engage in lessons that affirm diversity for all students.

## References

Blank, R. K. (2011). *Closing the achievement gap for economically disadvantaged students. Analyzing Change since no Child Left Behind using State Assessments and the National Assessment of Educational Progress.* Washington, DC: Council of Chief State School Officers.

Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching.* John Wiley & Sons.

Delpit, L. (2012). *Multiplication Is for white people: Raising expectations for other people's children.* New York, NY: The New Press.

Gay, G. (2002). Preparing for culturally responsive teaching. *Journal of Teacher Education, 53*(2), 106-116

Glaser, B. G. (1965). The constant comparative method of qualitative analysis. *Social Problems, 12*(4), 436-445.

Krull, K. (2000). *Wilma unlimited: How Wilma Rudolph became the world's fastest woman.* Houghton Mifflin Harcourt.

Hefflin, B. R. (2002). Learning to develop culturally relevant pedagogy: A lesson about cornrowed lives. *The Urban Review, 34*(3), 231-250.

Hefflin, B. R., & Barksdale-Ladd, M. A. (2001). African American children's literature that helps students find themselves: Selection guidelines for grades K-3. *The Reading Teacher, 54*(8), 810-819.

Hong, H. (1996). Effects of mathematics learning through children's literature on math achievement and dispositional outcomes. *Early Childhood Research Quarterly, 11*(4), 477-494.

Ladson-Billings, G. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education, 28*(6), 697-708.

Ladson-Billings, G. (1995). But that's just good teaching! the case for culturally relevant pedagogy. *Theory into Practice, 34*(3), 159-165.

Moyer, P. S. (2000). Communicating mathematically: Children's literature as a natural connection. *The Reading Teacher, 54*(3), 246-255.

Pérez, C. A. (2012). A story covers all: Using children's literature for acquiring cross-curricular competences in early childhood and primary. *1st International Conference: Teaching Literature in English for Young Learners*, 107-112.

Schiro, M. (1997). *Integrating children's literature and mathematics in the classroom: Children as meaning makers, problem solvers, and literary critics.* New York, NY: Teachers College Press.

Shatzer, J. (2008). Picture book power: Connecting children's literature and mathematics. *The Reading Teacher, 61*(8), 649-653.

Sleeter, C. E., & Grant, C. A. (2011). *Race, class, gender and disability in current textbooks. Politics of the Textbook.*

Tatum, A. W. (2006). Engaging African American males in reading. *Educational Leadership, 63*(5), 44.

Walker-Dalhouse, D. (1992). Using African-American literature to increase ethnic understanding. *The Reading Teacher, 45*(6), 416-422.

Whitin, P., & Whitin, D. (2004*). New visions for linking literature and mathematics.* Urbana, IL: The National Council of Teachers of English.

# ANALYSIS OF LARGER MEDIUMS FOR MENTAL MODELS OF SCIENCE AND SCIENCE TEACHING

Jenny Peters
Oklahoma State University
jenny.peters@okstate.edu

Stephanie Hathcock
Oklahoma State University
stephanie.hathcock@okstate.edu

*People's perceptions of scientists and science teaching are often measured using mental model tests such as the Draw-A-Scientist-Test (DAST) and Draw-A-Science-Teacher-Test (DASTT). Results generally showcase stereotypical images of scientists and teacher-directed images of science teaching. This research questions the traditional structure for these drawings by analyzing the difference in number of details and DASTT score when participants are presented with the typical half 8.5"x11" paper to draw on versus the full 8.5"x11" space printed on 11"x14"paper. Results showed that the larger drawing space produced more details while the overall DASTT score did not change. Implications for this result are discussed.*

## Introduction

Decades have seen the use of the Draw-A-Scientist-Test (DAST) to measure children's perceptions of scientists after the DAST was first created in 1983 (Finson & et al., 1995). In 1995, a companion checklist was developed called the DAST-C with the goal of making identification of particular details in the drawings more efficient and quantifiable (Finson & et al., 1995). More recently the research done by Finson, Beaver and Cramond has been extended to the development of the Draw-A-Science-Teacher-Test (DASTT) and the corresponding checklist (DASTT-C) which is used to quantify the details in drawings created by preservice teachers (PSTs) who plan to teach science at the elementary level (Thomas, Pedersen, & Finson, 2001). The goal of the DASTT-C is to assess the components of PSTs mental models with respect to science and science teaching. More specifically, the DASTT-C is intended to provide PSTs with a "reflective opportunity" (pg. 298) to imagine themselves as science teachers, identify their place on a teaching styles continuum, and consider how their science teaching beliefs developed (Thomas et al., 2001). Research suggests that teachers begin their careers with a teacher-centered approach and this combined with their stereotypical beliefs about science, scientists and science teaching can have negative impacts on student learning (Finson & et al., 1995; Thomas et al., 2001). Additionally, recent reform movements initiated by the National Research Council have sought to improve student perceptions of scientists by advocating science as a something students do, not something that is done to them (NRC, 1996). Consequently, the reform effort aligns the teaching of science with a student-centered inquiry-based approach where students are expected to be involved in "minds-on" (pg. 295) learning (Thomas et al., 2001). Therefore, the DASTT-C represents an opportunity to combat the negative effects by first providing a method of identifying and quantifying the mental models of science PSTs so that they might begin the process of moving away from a strictly teacher-centered approach to a more student-centered style.

The structure of the DASTT-C has not changed much over time and was inherited from the original DAST instrument. The DASTT-C has an area marked for drawing and a section below that for PSTs to provide a narrative of what is happening in their illustrations (Thomas et al., 2001). With the checklist portion, details representing a teacher-centered approach can be counted while the absence of the teacher-centered elements yields a score that aligns with a more student-centered approach (Thomas et al., 2001).

While many studies have employed the DAST to measure perceptions of scientists (e.g., Farland-Smith, 2012; Finson & et al., 1995; Thomas et al., 2001), fewer studies have used the DASTT-C (e.g., Ambusaidi & Al-Balushi, 2012; Thomas et al., 2001; Yilmaz, Turkmen, Pedersen, & Cavas, 2007). Additionally, none of these studies have questioned the traditional structure of the instruments even though research seems to indicate that modifications to the narrative prompts and directions can alter the details presented in the drawings (Farland-Smith, 2012; Finson & et al., 1995). The idea that modifying directions can change what participants draw is extended with this research study and applied to the drawing area or box that is provided for participants to use as their drawing space.

## Objectives of the Study

The purpose of this study is to describe and assess differences in PSTs mental model images as a result of the size of the area used for communicating their perceptions of teaching science. This research questions the traditional structure for these drawings by showcasing differences in PST's drawings when presented with a larger drawing space (full size of an 8.5"x11" area) printed on an 11"x14" medium. Research presented here will strive to answer two questions: 1) What are the differences in PSTs mental images when an entire 8.5" x 11" drawing space is used to communicate perceptions versus a the traditional drawing space that is about half of an 8.5" x 11" size paper is used, and 2) Will the characteristics and details provided allow researchers to better determine the perceptions PSTs hold with regard to science and science teaching?

## Related Literature

In her article *Development and Field Test of the Modified Draw-a-Scientist Test and the Draw-a-Scientist Rubric* (2012), Farland-Smith describes the unintentional limitations researchers place on their participants when they are too specific with their questions or prompts. Specifically, Farland-Smith compares her modification of the DAST instruction to simply “draw a scientist” to a more directed instruction in which participants were asked to include details about the scientists appearance, location and activity (Farland-Smith, 2012). While the participants in Farland-Smith’s study did as they were directed, she acknowledges the problem her modifications might have in terms of misrepresenting the true mental models held by her participants even though this was not her intention. The original DAST was “designed to capture students’ images of scientist regardless of writing ability” (pg. 111) and did not lead students into producing a drawing that was not representative of their true mental image (Farland-Smith, 2012; Finson & et al., 1995).

The present study was not intended to be a critique of the types of questioning or directions employed but rather an analysis of the effects of drawing a box or boundary to provide a drawing space for the participants. Most DAST and DASTT use approximately half of the page of an 8.5” x 11” sheet in which a thin lined rectangle is provided to mark the region to be used for drawing. This area is small (less than half the page) and the researchers wondered if this space was somehow unintentionally limiting the details provided by the drawing participants in the same way questions and instructions can limit the details provided in their responses.

## Methodology

The participants in this study were elementary PSTs enrolled in a science methods course. A goal of the methods courses was to help improve the mental models of PSTs to incorporate images of scientists and science teaching that are less stereotypical and more aligned with an inquiry-based approach to teaching science. The participants were mostly female (n = 104) with one male PST.

Data collection took place via administration of the DASTT-C instrument at both the start and conclusion of the science methods course. Participants were asked to draw themselves as a science teacher, and then they were asked to describe what the teacher and the students were doing in the drawing they just created. One group of PSTs was given the standard size DASTT-C instrument where both the drawing space and the narrative prompts are printed on the front of an 8.5”x11” sheet of paper. The drawing space is outlined by a thin black line that marks a rectangular box that occupies the top half of the page. Below the box the narrative prompts (“what is the teacher doing” and “what are the students doing”) are printed with lines to allow space for PSTs to write their responses. A second group of PSTs was given a modified version of the DASTT-C instrument where the drawing space was a rectangle whose area was 8.5”x11” printed on the front of an 11”x14” paper medium. The narrative prompts remain the same as the standard DASTT-C instrument but were printed on the back of the 11”x14” paper. Figure 1 shows an example of each type of drawing space.

For the research questions addressed in this study, the units of analysis were the details present in each of the drawings as well as the DASTT-C scores. Thomas, Pedersen and Finson (2001) outline the specific process to score drawings using the checklist. Using this process, each researcher scored the drawings, then compared and reconciled scores for validity and reliability. To accurately count the details present in each of the drawings, a DASTT-C Details Rubric was created based on a similar method used by Farland-Smith in her development of the modified Draw-a-Scientist Test (Farland-Smith, 2012). The DASTT-C Details Rubric contains four categories of details—types of words written on the drawing space, details describing the teacher, details describing the students, and details describing the environment of the class. All four categories were used to analyze a drawing to determine which details needed to be counted.

### Category 1: Word Details

Words were only counted as details if they were written on the drawing space and not in response to the narrative prompts that followed the drawing. There were several different types of words that were counted as details during analysis of the drawings. First, content words written in the drawing space were typically words that describe the content of the lesson being taught (i.e. “Chemistry” or “Plants and the Life Cycle”). Content word details were typically written on the whiteboards/smartboards that were also drawn in the space. Second, teacher action and student action words were also counted as details. These words were primarily written to describe what the teacher and the students were doing. For example, in Figure 1, the word “Facilitating” is written next to the teacher and the words “Experimenting” and “Researching” are written next to the students. These words indicate different actions for each individual in the drawing and therefore count as three separate details. The third type of words found in the drawings were label words. These words could range from simply labeling a desk or shelf to labeling a set of scribbles on the whiteboard as “directions” (Figure 1). Label words were only counted as details if they added to the interpretation of the drawing. If a desk in a drawing has sufficient detail to make it look like a desk, then the drawing evaluator could interpret the image as a desk without needing the label word “desk” on the drawing. Therefore, the label word “desk” (if present) would not count as a detail. It should be noted that undecipherable scribbles that are intended to look like writing (see Figure 1) are not counted as details because they add no more information other than to identify that a whiteboard has been used. However, if the scribbles are labeled, as they are in Figure 1 below, the label word would count as a detail. Lastly, the words that appear as teacher or student speech are counted as details if the wording indicates any information that contributes to the interpretation of the drawing. These types of words, typically appearing in a speech bubble or cloud, can list specific questions being asked by the teacher or by the students, and they can portray emotions and attitudes about science and science teaching.

### Category 2: Teacher Details

The teacher being present in the drawing counted as one detail. The original instruction to “draw a picture of yourself as a science teacher at work” elicited many PSTs to draw themselves with great amounts of detail including eyelashes and forehead wrinkles. However, some teachers were drawn as stick figures with happy faces and no other features. The DASTT-C takes into account several indicators for scoring the teacher image as it relates to the teaching continuum but this analysis was strictly concerned with the teacher figure being present or not. If the teacher was drawn, one detail was counted. Additionally, teacher motion in the classroom was considered a detail. If the participants drew the teacher’s walking pathway or movement by a dotted line, the line was counted as a detail.

### Category 3: Student Details

In general, the student details were limited to counting one detail for decidedly male and one detail for decidedly female student figures. Due to the assumption that the drawings were intended to be images of a classroom with many students, multiple students drawn were counted as one detail. For example, if several male students and several female students are drawn, one detail is counted for the males and one detail is counted for the females. At times, drawings included students who were each drawn in different clothes/appearance, doing different activities, or sitting (or standing) in different positions. For these cases, each student is counted as their own detail.

### Category 4: Environment Details

The fourth category in the rubric involves elements that are drawn as part of the environment surrounding the teacher and student figures. There were three main types of environments drawn—the classroom, the laboratory, and outdoors. Typical elements found in a classroom environment count as details if they are drawn in the drawing space. Items like student desks, which are all drawn the same, count as one detail. The activities/lesson materials/lab materials drawn on or around the student desks also count as one detail if they are all drawn to look the same. Some activities are drawn to represent different science stations or centers and those activities are counted only if they are indeed different. The teacher's desk or table counts as one detail and each item that is drawn on the desk is counted as a detail to account for multiple types of lessons that can happen in the classroom or in the lab. Whiteboards, smartboards and chalkboards are counted as single details if they are present in the drawing. Outdoor scenes were drawn by some of the PSTs and those environments would yield details as well. Rocks, trees, wildlife, water features, the sun, and clouds are all details that can be counted. Multiple versions are counted as one detail (i.e. one cloud or many clouds count as one detail).

Each drawing was evaluated according to the DASTT-C Checklist and the DASTT-C Details Rubric described above. The number of details per drawing were recorded and compared across the different size mediums. Figure 1 demonstrates the scoring of two drawings, one standard and one larger. According to the DASTT-C Details Rubric the total number of details in the standard drawing space was 16 while the total number of details for the larger drawing space was 9.

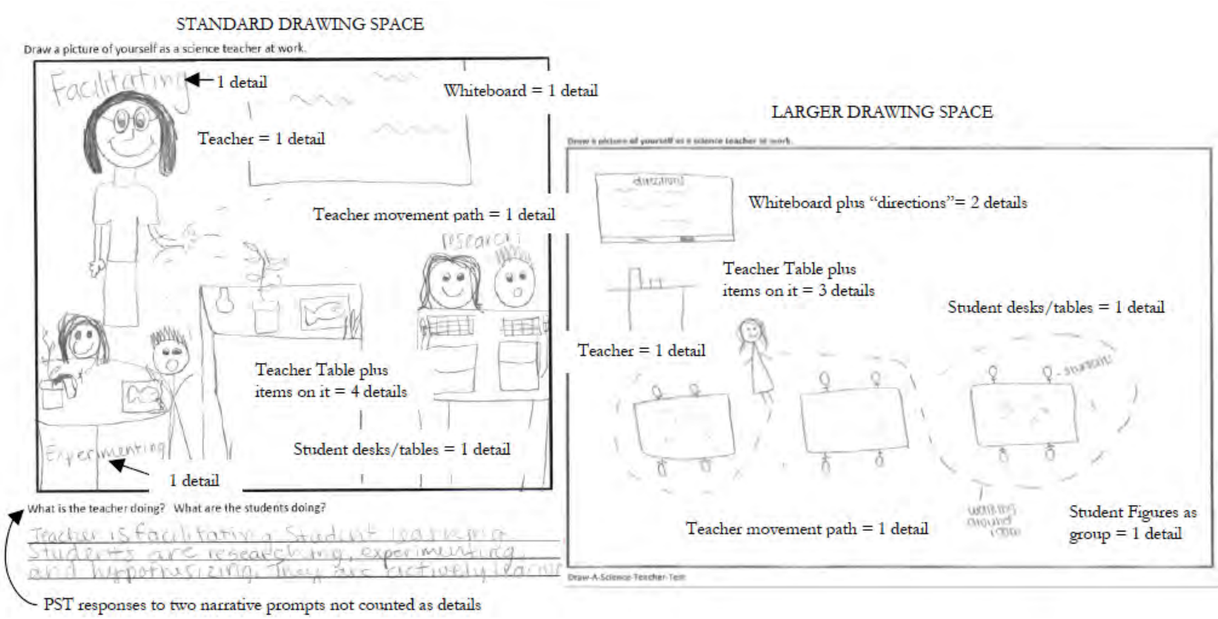



*Figure 1. Examples of drawings for both mediums with some details counted*

**Results and Discussion**

The independent samples t tests and Welch t' test used in this study assume a particular underlying theoretical population distribution bound by four conditions. The primary assumptions are that any sample obtained from the population will be independent and random samples that are distributed normally while maintaining equal variances. These assumptions were verified and a few violations did occur. Histograms of the data showed some minor nonnormality for all sets of data associated with the standard size medium and the larger size medium. However, even with these slight departures from normality, the effects on Type I and Type II errors were minimal given the use of a two-tailed test (Lomax, 2012). Homogeneity of variance was assumed in each test except the comparison between mean number of details for the standard medium and the larger medium for the time period corresponding to the beginning of the science methods course. Finally, the assumption of independence was likely not met due to the lack of random assignment of the participants' drawings to levels represented by the standard and larger mediums.

Independent samples t tests were conducted to determine if the mean number of details and the mean DASTT-C scores differed between the standard size medium and the larger size. The DASTT-C scores for both mediums and time periods showed no statistically significant difference. Table 2 shows the mean number of details and standard deviations for each time period with respect to the drawing space size.

Table 2

*Number of Details in the Standard vs. Larger Drawing Space*

| | Standard [a] | | | Larger [b] | | |
|---|---|---|---|---|---|---|
| Time Period | *n* | *M* | *SD* | *n* | *M* | *SD* |
| Course Beginning [c] | 71 | 6.085* | 1.842 | 34 | 8.088* | 3.019 |
| Course Ending | 70 | 6.986* | 3.246 | 32 | 8.656* | 3.033 |

*Note*. [a] Approximately half of 8.5"x11" paper; [b] Full 8.5"x11" paper;
[c] Means tested by the Welch t' Test; * $p < .05$

In the case where equal variances could not be assumed, the data was analyzed using the Welch t' Test (Lomax, 2012) that is appropriate when the population variances are unequal and the sample sizes are unequal. As Table 2 shows, the Welch t' test indicated the mean number of details (course beginning) were statistically significantly different ($t = 3.565$, $df = 45$, $p = .0004$). Thus, the null hypothesis that the mean number of details were the same between the different size mediums was rejected at the .05 level of significance. More specifically, more details on average were found with the larger drawing medium than with the standard size area. Likewise the independent t-test comparing the number of details for the course ending time period shows the mean number of details were statistically significantly different ($t$ = 2.461, $df$ = 100, $p$ = .0078). Again, the analysis showed that for this period, more details on average were found with the drawings completed using the larger medium.

## Implications

The lack of a statistically significant difference between the DASTT-C scores for the larger medium as compared to the smaller medium suggests that the checklist is a reliable measure of PSTs mental models of science teaching regardless of the size of the drawing area. This finding is consistent with past research studies that have employed the DASTT-C (Thomas et al., 2001). However, the significant difference found with regard to the number of details provided in the drawings with the larger medium indicates that the traditional structure of the DASTT-C might be limiting the participants and prohibiting them from fully expressing their complete ideas. Similar to the Farland-Smith study (2012), this limitation is unintentional but the finding in this study warrants a future probe into the size of the mediums on which PSTs must draw. More appropriately, the additional details afforded by the larger drawing space can serve to contribute to and confirm an evaluator's understanding of the images. If the evaluator can form a more complete understanding of what the details in the drawings represent, it follows that the DASTT-C score will be a more accurate measure of the image. While there is no one correct way to draw oneself as a science teacher, the intended purpose of the DASTT-C is to allow the PSTs an opportunity to reflect on their position on a teaching styles continuum. If they are not given the space to completely represent their mental models, their reflection period will be limited to considerations made with regard to incomplete data.

## References


Ambusaidi, A. K., & Al-Balushi, S. M. (2012). A Longitudinal Study to Identify Prospective Science Teachers' Beliefs about Science Teaching Using the Draw-a-Science-Teacher-Test Checklist. *International Journal of Environmental and Science Education, 7*(2), 291-311.

Council, N. R. (1996). National science education standards. Washington, DC: National Academy Press.

Farland-Smith, D. (2012). Development and Field Test of the Modified Draw-a-Scientist Test and the Draw-a-Scientist Rubric. *School Science and Mathematics, 112*(2), 109-116. doi:10.1111/j.1949-8594.2011.00124.x

Finson, K. D., & et al. (1995). Development and Field Test of a Checklist for the Draw-a-Scientist Test. *School Science and Mathematics, 95*(4), 195-205.

Lomax, R. G., and Hahs-Vaughn, D. L. (2012). *An Introduction to Statistical Concepts* (3rd ed.). New York, NY: Routledge.

Thomas, J., Pedersen, J., & Finson, K. (2001). Validating the Draw-A-Science-Teacher-Test Checklist (DASTT-C): Exploring Mental Models and Teacher Beliefs. *Journal of Science Teacher Education, 12*(4), 295-310. doi:10.1023/A:1014216328867

Yilmaz, H., Turkmen, H., Pedersen, J., & Cavas, P. H. (2007). Evaluation of pre-service teachers' images of science teaching in Turkey. *Faculty Publications: Deparment of Teaching, Learning and Teacher Education*(Paper 80).

# EFFECTS OF A SUMMER PROGRAM FOR UNDERSERVED ELEMENTARY CHILDREN ON MATHEMATICS LEARNING

Elaine Tuft
Utah Valley University
Elaine.Tuft@uvu.edu

*Many are concerned with potential learning loss that can occur during the summer break. This is of particular concern for underserved populations of elementary school children. This paper describes a summer school program that was designed to serve this population as well as its effects on the retention and learning of mathematics knowledge. The researchers found that, on average, students in each grade level were able to maintain or improve their performance on mathematics tests that assessed knowledge related to the number and operation concepts that were designated by the state core standards for their corresponding grade.*

## Introduction

Summer learning loss, especially among elementary students, is an ongoing concern to many educators. The potential loss of knowledge that was gained during the previous elementary school year is of particular worry with regards to the children of those who are in a low socio-economic class. This is because their parents may not have time or money to provide experiences that might help mitigate that loss. Fortunately, many educators have developed and provided summer programs that have been successful in lessening the loss of student achievement. Knowing the potential of these types of programs, a group of university professors from a School of Education; the school, community, and university partnership at that university; and the United Way collaborated to submit a proposal for a grant to provide a summer school opportunity for elementary students in an underserved section of a moderately large city. This proposal was submitted in response to an RFP from the state's Department of Workforce Services. This proposal was funded to conduct the program for three years.

This summer school program was held for eight weeks in a small community center that was situated in the part of the city where the target population lived. Classes were held Monday through Thursday from 8:00 until 1:00. The students were served both breakfast and lunch. Each student had classes daily in mathematics, literacy, arts enrichment, and physical education. An employee from the local library also came and did story time with them once a week. In addition, they went on three field trips during the program to various local interactive museums.

The students were divided into three groups based on the grade they had just finished and rotated through each class. The youngest group consisted of students who had just finished kindergarten or first grade. The students in the second group had just finished second or third grade. The students in the oldest group had just finished fourth, fifth, or sixth grade. Teachers of the core classes were licensed, experienced, currently-practicing elementary school teachers who had all had taught at least four years. There were two teachers for literacy, one for mathematics, and one for arts enrichment. The mathematics teacher had an elementary mathematics endorsement, a master's degree in Curriculum Design and Instruction, and eight years of teaching experience. Professors of Education from the university that received the grant with specialties in mathematics, literacy, and arts education helped the teacher who taught in their area of expertise plan and prepare for instruction during the program. There were also six mentors who helped with the program, assisting the teachers and helping with behavior management. Two of the six mentors had recently completed the Elementary Education program the participating university, and they were going to be teaching full-time in the fall. Two of the mentors were current students in the Elementary Education program, and the final two were students majoring in counseling and social work. Two mentors were assigned to work with the youngest group, and the other two groups of students each had one mentor. One mentor was assigned to assist the program coordinator, and the other helped with physical education and by giving needed support to other groups.

## Purpose of the Study

The purpose of this study was to assess the effects of this summer school program on the learning and retention of mathematics knowledge of the participants. Although each student in the summer program participated in mathematics, literacy, and arts enrichment classes, the purpose of this study was to specifically look at the mathematics learning.

## Related Literature

### Summer Learning Loss

McCombs, Augustine, Schwartz, Bodilly, McInnis, Lichter, and Cross (2011) reported studies that showed that after summer vacation, on average, students performed roughly one month behind where they had performed in the spring. The loss was especially severe in mathematics. Other research on the loss of academic learning during the summer has shown that without ongoing opportunities to learn and practice essential skills, students fall behind on measures of academic achievement during the summer months, losing as much as two months of grade-level equivalency in mathematical computation (Alexander, 2007; Cooper, 2003; McLaughlin & Smink, 2009). Even more concerning, still other research has suggested that this summer learning loss is cumulative. When students have repeated episodes of learning loss, it results in them falling further and further behind (McCombs et al. 2011).

### Purpose of Summer School Programs

Summer school programs have been created to help lessen that potential learning loss. However, that hasn't always been the case. Originally, many summer school programs were created with the purposes of remediation or prevention of behavior problems (Cooper, Charlton, Valentine, & Muhlenbruck, 2000). In recent decades, there has been a change in thinking about the role of summer school programs. Instead of the punitive and remedial models of the past, summer school programs are now thought of as an opportunity for a blend of core academic learning with other experiences in the arts, sports, skill-building, and building meaningful relationships (Cooper et al., 2000; McLaughlin & Smink, 2009).

### Effectiveness of Summer School

Evidence suggests that summer school programs can lessen the drop in achievement over the summer break (Borman & Dowling, 2006; Cann et al., 2014; Lauer, Akiba, Wilkerson, Apthorp, Snow, & Martin-Glenn, 2006). In a meta-analysis of 33 out-of-school-time programs conducted by Lauer et al. (2006), they found small but statistically significant positive effects of out-of-school-time programs on mathematics student achievement. They found that whether the out-of-school-time program took place after school or during the summer did not make a difference in effectiveness (Lauer et al., 2006). When summarizing findings from a meta-analysis conducted by Cooper, Nye, Charlton, Lindsay, and Greathouse (1996), they recommended that the research on alleviating summer learning loss suggested that a primary focus on mathematics instruction seemed to be the most needed (Cooper et al., 2000). Cooper et al. (2000) also concluded that summer school was an effective system for achieving educational goals, and while the benefits of such programs vary due to the differences in the children and the content and delivery of the program, overall, the positive results were unmistakable.

### Summer Programs and Low SES Children

Considerations of the needs of children from low SES families or other underserved populations have been a major influence on the creation of summer programs, and several pieces of research have shown that there are distinct differences in the rates at which low-income and higher-income students learn in the summer (Cooper et al., 2000; Lauer et al., 2006; McCombs et al., 2011). Some studies have shown that while students from high-income and low-income families learn at the same rate during school, learning for the low-income students falls far behind during the summer months. This is one of the reasons that some have suggested that summer programs for low SES children are especially important (McCombs et al., 2011).

## Methodology

### Participants

The participants in this study were the K-6 students who participated in the summer program. The majority of these students live in an area of the city that is considered to be of low socio-economic status, includes many apartment buildings, and is almost entirely composed of non-white citizens. There were a total of 61 students enrolled in the program. Of those 61 students, 54 attended two or more times. Thirty-five attended more than 20 days. Of the 54 who attended at least twice, 51 were Latinos/as, and the other three were Caucasian. Twelve of the students were not proficient in English. Two students had been diagnosed with autism, and one of those students had also been diagnosed with ADHD. Parents of the participating students filled out a questionnaire with demographic information at the beginning of the program, and 23 reported that their children received free and/or reduced rate lunch; five reported that they did not; and 26 did not report. Most of these parents had more than one child in the program. Thirty-seven students were given both pre and posttests that were used in the analyses, 31 of those students attended more than 20 days.

### Instruments and Data Collection

Information about each child's understanding of mathematics concepts was collected through mathematics tests that were given to the students the first or second day of the program and again during the last week of the program. The students were given a test that corresponded to the grade they had just completed. A different test was created for each grade K-6 by the researchers. The researchers created tests for grades 2-6 by using a bank of questions from the appropriate grade-level test from *Go Math* (Houghton Mifflin Harcourt, 2011), which was the mathematics curriculum that was used by the participating school district. The mathematics emphasis of this summer program was number and operation. Therefore, representative items were chosen that matched the grade-level state core standards in those areas. These tests each consisted of 10 items. The test items for Kindergarten and 1$^{st}$ grade were created by the researchers, also to correspond with the state core standards related to number and operation in those grades. The Kindergarten and 1$^{st}$ grade tests were administered individually by the university professor and a trained student mentor. The Kindergarten test consisted of nine separately scored points; the first grade had seven; and the tests for 2$^{nd}$ – 6$^{th}$ grade each consisted of 10 items.

Qualitative data was also collected to assess the students' retention and learning in mathematics through observations, anecdotes, informal interviews with students, and documented patterns of learning.

### Analysis

The quantitative analysis of the mathematics learning and retention of these students was mostly descriptive. Pre and post means on the mathematics tests were compared for each grade as a total score and by individual item. Because of the small number of students in each grade, it was deemed that making comparisons using *t*-tests was not meaningful. The qualitative data was analyzed by looking for themes and patterns.

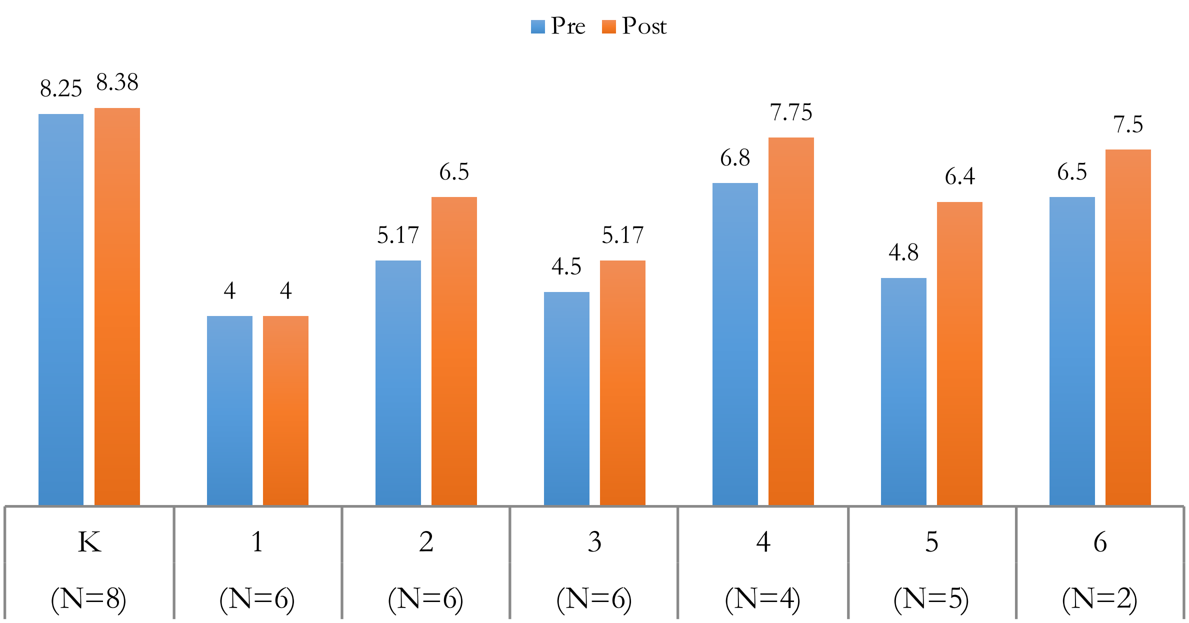
## Results and Discussion

Both quantitative and qualitative analyses of the data used to inform the effectiveness of this program in helping these children retain or increase their mathematics learning indicated that the effect had been positive.

### Quantitative Analysis

Of the 37 students who took both the pretest and posttest, 31 of them attended more than 20 days. Of those 31, the scores for 20 of them increased, six of them were the same, and five decreased. Overall, the average score for each grade increased or stayed the same. Figure 1 shows the comparison between the pretest and posttest

*Figure 1: Pretest and Posttest Comparison by Grade*



Qualitative Analysis

Qualitative experience and analysis of this program also showed that it had positive effects on the students' understanding of and engagement with mathematics. For example, throughout the program, the mathematics teacher spoke with students, and they expressed their feelings of the mathematics class and the program. Several of the students told the mathematics teacher, "I used to hate math, but now it is one of my favorite subjects." One student usually rode with another family to and from the program. However, one day, the family that was giving the ride was not going to be attending, and the student's mother told the mathematics teacher, "My boy did not want to miss math class that day. So, he got up and walked about two miles that morning, all so he could get to your class. I don't know what you are doing, but he used to dislike math, and now he loves it."

The actions and comments of each of the groups were also indicative of the success of the mathematics classes. The oldest group had the math class first, and on many occasions, the students would be so engaged in their assignment, they would ask if they could stay in for recess to finish their assignment. The youngest group came to math right before lunch. In the beginning of the program, well before lunch time, the majority of the students would start complaining about being hungry and ask when they were going to have lunch. This was a message to the math teacher that the students were losing interest in the lesson. The teacher tried to employ many different teaching methods to get the students more involved. Towards the end of the program, success was finally reached when the teacher would tell the students it was time for lunch, and they would say things like, "Oh man, can we work on this just a little bit longer?" That same group was able to progress from only being able to count to 75 as a group, to being able to count to 130. There was also visible success with the middle group. They were the last class of the day, and after the first little while, they were always excited to go to class. In the beginning of the program, this group had a difficult time understanding how to decompose numbers to help with addition and subtraction, and at the end of the program, they could decompose numbers. This helped their fluency in adding and subtracting numbers improve.

## Implications

The effectiveness of this summer program has great educational importance. The analyses showed that it was effective in diminishing the loss of mathematics achievement of these underserved students during the summer. It also showed that this type of program can be valuable in helping students improve their attitudes toward mathematics, which affects their achievement. Therefore, it can be implied from the results of this and other studies that summer school programs can be an effective and worthwhile investment of resources and time in helping children alleviate potential summer learning loss.

## References

Alexander, K. L. (2007). Lasting consequences of the summer learning gap, *American Sociological Review*, *72*, 167-180.

Borman, G. D., & Dowling, N. M. (2006). The longitudinal achievement effects of multi-year summer school: Evidence from the Teach Baltimore randomized field trial. *Educational Evaluation and Policy Analysis*, *28*, 25-48.

Cann, D., Karakaplan, M., Lubke, M. & Rowland, C. (2014, February) *Assessing the effectiveness of New Mexico's K-3 plus summer learning initiative*. Evanston, IL: Society for Research on Educational Effectiveness.

Cooper, H., (2003). Summer learning loss: The problem and some solutions. LD Online: The Educators' Guide to Learning Disabilities and ADHD. Retrieved from http://www.ldonline.org/article/8057/.

Cooper, H., Charlton, K., Valentine, J. C., & Muhlenbruck, L. (2000). Making the most of summer school: A meta-analytic and narrative review. *Monographs of the Society for Research in Child Development* (Serial No. 260), *65*(1), 1-118.

Cooper, H., Nye, B., Charlton, K., Lindsay, J., & Greathouse, S. (1996). The effects of summer vacation on achievement test scores: A narrative and meta-analytic review. *Review of Educational Research*, *66*, 227-268.

Houghton Mifflin Harcourt Publishers. (2011). *Go math!* Boston: Author.

Lauer, P. A., Akiba, M. Wilkerson, S. B., Apthorp, H.S., Snow, D., & Martin-Glenn, M.L. (2006). Out-of-school-time programs: A meta-analysis of effects for at-risk students, *Review of Educational Research*, *76*(2), 275-313.

LeBoff, B. A. (1995). *The effectiveness of a six-week summer school program on the achievement of urban, inner-city third-grade children* (Doctoral dissertation, Texas Southern University, 1995). Dissertation Abstracts International, *56*, 10A.

McCombs, J. S., Augustine, C. H., Schwartz, H. L., Bodilly, S. J. McInnis, B., Lichter, D. S., & Cross, A. B., (2011) *Making summer count: How summer programs can boost children's learning*. (Report commissioned by The Wallace Foundation). Santa Monica, CA: the RAND Corporation.

McLaughlin, M. P. P. & Smink, J. (2009). Summer learning: moving from the periphery to the core. Denver, Colorado: *Education Commission of the States. Progress of Education Reform*, *10*, 3.

## Acknowledgements

This program was funded by a grant from the STATE Department of Workforce Services.

# THE INFLUENCE OF INTERDISCIPLINARY CO-PLANNING TEAMS ON BELIEFS OF SECONDARY MATHEMATICS AND SCIENCE TEACHERS

Michelle Cetner
North Carolina State University
mcetner@ncsu.edu

Allison McCulloch
North Carolina State University
awmccull@ncsu.edu

*The Interdisciplinary Co-planning Team (ICT) model was developed to support teachers to communicate regularly and intentionally about the connections between mathematics and science in order to help students build interdisciplinary connections. The model was designed to mitigate common obstacles to interdisciplinary teaching that have been identified in research. Here we report on a case study of 3 pairs of secondary mathematics and science teachers who implemented the ICT model over the course of 10 weeks. Findings related to influence of the co-planning process on teacher beliefs regarding the nature of each subject, teaching and learning, making interdisciplinary connections are discussed.*

## Introduction

Mathematics and science share a close relationship in the development of our modern world but are often taught separately in schools. Teachers can emphasize the relationship between these subjects to help students gain deeper understanding of the content and critical thinking skills as well as increase engagement in the classroom (e.g., Becker & Park, 2011; Czerniak et al., 1999). Despite research that reports such successes, efforts to connect these subjects have been mostly local or limited because of obstacles such as teacher experience, knowledge, tradition, comfort levels, and the structure of school itself (Moore & Smith, 2014). The *Interdisciplinary Co-planning Team* (ICT) model was created by the researchers to guide teachers to work within the existing school structure and overcome obstacles that have previously limited or discouraged interdisciplinary teaching.

In the ICT model, teachers of different subjects are paired and participate in co-planning sessions on a weekly basis to discuss concepts and practices from each other's content areas and how connections can be drawn between them. They look for and create examples and activities that would help students to develop conceptual understanding and consider practices from both subject areas. After co-planning, the teachers implement their plans in their own classrooms. They then meet to reflect on their lessons, discuss student understanding, and co-plan future lessons.

## Objectives of the Study

This paper reports on part of a larger study, in which the ICT model was implemented with 3 pairs of teachers (two high school pairs and one middle school pair) through a professional development intervention. Here we focus on the changes in beliefs for each teacher pair that occurred as a result of the intervention. Based on extant literature, these beliefs are separated into three categories to address the following research question: in what ways does participation with the ICT model influence teachers' expressed beliefs regarding a) the nature of mathematics and science; b) teaching and learning; and c) making interdisciplinary connections?

## Literature and Framework

Beliefs are a complex, but relatively stable system of contextually based personal knowledge, which inform thoughts and behavior (Llinares, 2002). They are formed by personal and cultural experiences (Philipp, 2007), and are organized in interrelated clusters (Llinares, 2002). Most literature concerning math and science teacher beliefs falls into the categories of beliefs relating to the nature of mathematics and science, and beliefs about teaching and learning.

Regarding the nature of mathematics and science, several related frameworks have been established placing teachers on a continuum from authoritarian to problem solving beliefs about mathematics (e.g. Amirali & Halai, 2010), and traditional to inquiry based views of science (e.g. Mansour, 2013). Traditional teacher beliefs include that math is rules and computation (e.g. Cross, 2009), and that science is an objective body of knowledge produced by a rigid and universal scientific method (e.g. Wallace & Kang, 2004).

Teacher beliefs about teaching and learning are intertwined, and claims about them are more varied and complex than beliefs about the nature of mathematics or science, especially when planning for instructional tasks (Wallace & Kang, 2004), partly because of the focus on human activity (Turner, Christiansen & Meyer, 2009). Teachers tend to focus on building student confidence (Andrews & Hatch, 2000), transferring pedagogical authority (Wallace & Kang, 2004), and inducting students into widely accepted ways of thinking (Turner, Christiansen & Meyer, 2009). Teachers often have apparent inconsistencies between their espoused and enacted beliefs (e.g., Handal, 2003), which can usually be explained by realizing multifaceted nature of beliefs, which include teacher ideals, student abilities, and administrative expectations (Barkatsas & Malone, 2005).

Teacher beliefs and planning practices form a complex relationship with each other and with actual teaching practices. Planning is a complex and ongoing mental dialog, which is influenced by teacher's beliefs (e.g., Wilson & Cooney, 2002). Co-planning, as well as teaching and other experiences influence teacher beliefs, particularly if outside of the teacher's norm (Llinares, 2002).

The literature regarding the relationship between beliefs and planning informed the design of the conceptual framework for this study (see Figure 1). The framework expresses the reciprocal relationship that exists among teacher beliefs, planning, plans for teaching, and teaching outcomes. Teacher beliefs form an umbrella under which all actions and plans are mediated. The solid arrows represent components that research has shown directly influence each other. The dashed arrows signify influence that occurs through reflection. All of this is set against the backgrounds of teacher factors and external factors.

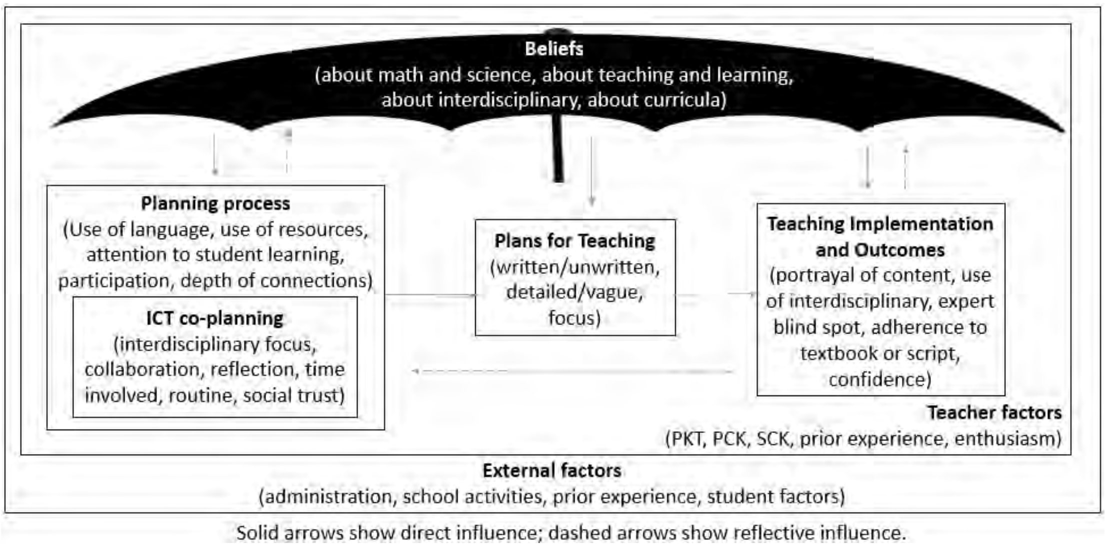



*Figure 1*. *Conceptual co-planning framework*

**Methodology**

This study employed a qualitative embedded multiple-case intervention design in order to gather and examine data regarding the nature of teachers' beliefs as they related to the planning processes and plans during implementation of the ICT model.

Participants and Intervention

The participants were a convenience sample of three pairs mathematics and science teachers, with each teacher pairing constituting one case. The first pair, M and S, are both experienced 8$^{th}$ grade teachers at a middle school in a small southeastern city. (Note: For each pair the teacher initial that is first alphabetically denotes the mathematics teacher in the pair.) J and W are both novice teachers, to mostly 9$^{th}$ grade students at a rural southeastern high school. K and T are both in their fifth year of teaching, to mostly 10$^{th}$ grade students at the same high school. Aside from W, who co-taught a World Dynamics class that blended English, History, and Earth Science, none of the teachers had previous sustained experience collaborating with teachers of other subjects.

The cases in this study are bounded in time by the intervention, a 10-week implementation of the ICT model with teachers. The intervention took an alternating, whole group - teacher pair approach, in which the teacher pairs were introduced to the model as a whole group, and then spent several sessions co-planning alone with their partner teacher and the researcher, who acted as facilitator. Midway through (at week 5) and at the end (week 10), the whole group convened to debrief and troubleshoot. Debriefing and troubleshooting were guided by predetermined group discussion questions.

### Data Sources and Analysis

The primary data source for teacher beliefs were individual semi-structured pre- and post-intervention interviews held with each teacher. Other data sources, including observations of co-planning sessions, teacher reflections, and artifacts (i.e. lesson plans, worksheets) were collected as a part of the larger study, and reviewed to determine shifts in teacher beliefs as well. All interviews and co-planning observations were transcribed verbatim in their entirety. Interviews were coded for beliefs about the nature of mathematics and science, beliefs about teaching and learning, and beliefs about interdisciplinary connections. Afterward, all other data sources were reviewed and coded for evidence to corroborate the beliefs that teachers espoused in interviews. Codes were sorted and reviewed for each case before cross-case comparisons were completed, using a constant comparative method of analysis (Walker & Myrick, 2006).

## Results and Discussion

During the course of the intervention, all three teacher pairs participated in regular co-planning sessions, implemented several co-planned lessons in their classes, and experienced shifts in beliefs in multiple areas. This paper will focus on the most profound shifts for each pair to provide an example of how the ICT model can influence beliefs: J and W about the nature of mathematics and science; M and S about teaching and learning; and K and T about interdisciplinary connections.

### The Case of J and W – Beliefs about the Nature of Mathematics and Science.

Both W and J were relatively unchanging in their beliefs about their own subjects over the course of the intervention. J firmly held a belief that mathematics is “logical process” and “steps with reasons,” as he repeated several times, both during interviews and co-planning sessions. In mathematics, he emphasized that students should “look for the big picture.” W believed firmly that science is “exploration and inquiry” and “understanding how the world works," and that progress meant "changes in theory." However, through interaction with each other and their content, both dramatically changed their views about the opposite subject.

W initially viewed mathematics a series of memorized facts and procedures. In trying to describe math initially, she said, "Analytical? Right brained?" She also said that in order to be good at math, students need to practice, and offered her own students many repeated practice problems when her physical science class included "math" topics. Her discomfort with mathematics was evident in her planning and teaching of physical science. Especially toward the beginning of the intervention, she stated that she did not encourage higher order mathematical thinking in her class, and became noticeably agitated when her partner teacher would begin to discuss concepts such as interpreting graphs. In contrast, during her post-intervention interview she said, "I was like, wow, math really is useful, if you connect it to these real-world situations. If you connect math to what they're doing in science, to me it makes it much more relevant." J's transition was marked more by his knowledge of science. He based his initial beliefs about the nature of science on his understanding of mathematics. "I see math and science as kind of siblings. Math is the reasoning where science is the application." While J did not abandon this view, as he did change his view of what application looks like. In his post-interview, he said, "the data you get from the lab will help you reach a conclusion and see if you need to test again, or see if you need to revise your hypothesis." Along with seeing science as more flexible, J mentioned an underlying structure and background knowledge that was absent from his earlier descriptions.

J and W's shifts in beliefs about the nature of mathematics and science may be because they are beginning teachers, and still forming ideas their ideas, especially as they relate to teaching in their classes. W initially believed in a particularly authoritarian view of mathematics (Andrews and Hatch, 2000), which shifted over the course of the intervention to include more constructivist and pragmatic views (Handal, 2003). It was obvious that W brought her own performance anxiety with her, which has been shown to enhance a desire to protect students from difficulties that mathematical problems might present (Perkkila, 2001). This gradually dissipated as she worked with J and gained more skill, understanding, and confidence. In the same way, J began to broaden his view from a rigid scientific method (Wallace & Kang, 2004) to a more flexible approach to science.

### The Case of M and S – Beliefs about Teaching and Learning.

Both M and S based the content of their classes on state standards, and they both believed that a part of their responsibility was to create experiences for students that would help them connect their knowledge to the outside world. For example, S said during her interview, “Some activities, it’s just to give them exposure to a concept. Like, when we go outside, pointing things out to them, you’re giving them kind of some mental connections that they can kind of refer back to,” and during a co-planning session, “the children that have the most difficult time have the least amount of experience in life.” Because of an emphasis on testing at this school, often these experiences were thought of as something to add on to lessons, and not an integral part of the lesson itself, so they were easily discarded when time became short.

Initially interdisciplinary connections were in this same category, add-ons that were often discarded. Both teachers would search for a way to create a big project that connected mathematics and science, and then determine that they did not have time to implement it. However, as co-planning sessions progressed, they began to find connections that they could implement within their lessons more frequently without adding additional time. During her final interview, M commented that making such connections is “not just thinking of it as something extra, but this is a part of what I do now. So that mindset has definitely changed. I think that’s the biggest part of it. Like, I mean, when I went to S at first, it was like what can I do extra, but now this is what I’m going to do.”

M and S’s shift in the area of teaching and learning exemplify one way that teachers may exhibit apparent inconsistencies in their espoused beliefs about teaching and learning (Barkatsas & Malone, 2005). Both of these teachers initially held a problem solving view of learning (Handal, 2003; Perkkila, 2001), but because they perceived an administrative emphasis on testing scores, they viewed their active learning experiences as an addition to lesson plans instead of part of them, and they were easily discarded, as seen by Wallace & Kang (2004). Throughout the intervention, creating interdisciplinary experiences became a part of what they did, rather than an additional task, and they were able to resolve inconsistencies between espoused and enacted beliefs.

### The Case of K and T – Beliefs about Interdisciplinary connections.

Both K and T stated several times that math is the language of science. K found that the most convenient way to draw connections was through word problems in his math class, and T found the same with her science labs. Word problems and labs were both discussed thoroughly during co-planning sessions, and each teacher gave suggestions to the other about how their content could be enhanced through the other’s venue. Over the course of the intervention, T realized that she could enhance a lab simply by bringing it to K to look for connections, and that it is not necessary to have a predetermined question before approaching the mathematics teacher. She impressed herself several times saying, “I think that activity was meaningful, and an extension of what I wanted them to learn.” By the midpoint of the intervention, T commented to the entire group the value of “just having someone with a different expertise that can just lend you suggestions that I think I would otherwise miss.” During her final interview she noted:

> I think this is what I’ve really learned from the process. You don’t have to come with this big, huge idea of how you’re going to change everything with this big old project. I think actually that’s an unfortunate thing to do. I think it’s more valuable if you just come to a meeting and say this is what we are learning about. Can you think of any way that you could incorporate an idea very simply.

K and T exemplified a shift in their use of interdisciplinary connections that most teachers experienced to some degree. They began with a focus on impractical “big projects” and tried to have connections already in mind when talking to each other. This focus on activity, such as was described by Turner, Christiansen & Meyer (2009), was actually a hindrance in this context. As K and T realized that it was more useful to use activities as a context to discuss content in detail, they were able to use each other’s expertise to find more connections and more relevance.

## Implications

Although it has been shown that creating interdisciplinary experiences for students has many positive effects (Becker & Park, 2011), positive examples in literature are few, partly because of obstacles mentioned above (Czerniak et al., 1999). This case study demonstrates through three examples that participation with the ICT model encourages teachers to form a sustainable, collaborative routine in which they discuss content and connections and use these discussions to form interdisciplinary experiences for their students. The teachers in this study gained the knowledge and confidence needed to implement interdisciplinary experiences with their students (case of J and W), overcame internal conflicts in beliefs about teaching and learning (case of M and S), and they broadened their view of collaboration and interdisciplinary connections to include a variety of experiences (case of K and T), creating a sustainable experience that each of the teachers in the study valued and desired to repeat in the upcoming school year. Furthermore, although all three teacher pairs were initially hesitant about giving up an hour or more each week, each commented on the value of the time spent during their later interviews, and said that they will pursue co-planning the following school year.

Although the three cases in this study do not permit generalizability, schools and teachers may be encouraged to follow the ICT model, whose more intimate pairings may aid in the creation of this social trust, which may lead to increased knowledge and confidence, as well as facilitate scheduling regular meetings to continue the process. This then becomes a part of the teacher's routine, instead of an additional box to check on an already growing list. The ICT model encourages teachers to use each other's expertise to create and implement frequent interdisciplinary experiences for students, benefitting both student and teacher.

This paper reported on part of a larger study, which analyzes the co-planning process as well as the plans that the teachers made. In addition to this analysis, future research concerning the ICT model includes implementation with more teacher pairs, in a variety of schools, and for a more prolonged period of time. The researchers expect to support the growth of many teachers through dissemination of the ICT model.

## References

Amirali, M., & Halai, A. (2010). Teachers' knowledge about the nature of mathematics: A Survey of secondary school teachers in Karachi, Pakistan. *Bulletin of Education and Research*, *32*(2), 45-61.

Andrews, P., & Hatch, G. (2000). A comparison of Hungarian and English teachers' conceptions of mathematics and its teaching. *Educational Studies in Mathematics*, *43*(1), 31-64.

Barkatsas, A. T., & Malone, J. (2005). A typology of mathematics teachers' beliefs about teaching and learning mathematics and instructional practices. *Mathematics Education Research Journal*, *17*(2), 69-90.

Becker, K. H., & Park, K. (2011). Integrative approaches among Science, Technology, Engineering, and Mathematics subjects on students' learning: A meta-analysis. *Journal of STEM Education: Innovations and Research*, 12, 23-37.

Cross, D. I. (2009). Alignment, cohesion, and change: Examining mathematics teachers' belief structures and their influence on instructional practices. *Journal of Mathematics Teacher Education*, *12*(5), 325-346.

Czerniak, C. M., Weber, W. B., Sandmann, A., & Ahern, J. (1999). A literature review of science and mathematics integration. *School Science and Mathematics*, 99(8), 421-430.

Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator*, *13*(2), 47-57.

Llinares, S. (2002). Participation in reification in learning to teach: The role of knowledge and beliefs. In G. Leder, E. Pehkonen, G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 195-210). Netherlands: Kluwer.

Mansour, N. (2013). Consistencies and inconsistencies between science teachers' beliefs and practices. *International Journal of Science Education*, *35*(7), 1230-1275.

Moore, T. J., Smith, K. A. (2014). Advancing the State of the Art of STEM Integration. Journal of STEM Education, 15(1), 5-10.

Perkkila, P. (2001). Primary school teachers' mathematics beliefs, teaching practices, and use of textbooks. In M. Ahtee, O. Bjorkqvist, E. Pehkonen, V. Vatanen (Eds.)*, Research on Mathematics and Science Education* (pp. 37-46). Jyvaskyla: University Printing House.

Philipp, R. A. (2007). Mathematics teachers: Beliefs and affect. *In: F. K. Lester (Ed.): Second Handbook of Research on Mathematics Teaching and Learning. Charlotte, NC: Information Age Publishing*, 257-315.

Turner, J. C., Christensen, A., & Meyer, D. K. (2009). Teachers' beliefs about student learning and motivation. In *International handbook of research on teachers and teaching* (pp. 361-371). Springer US.

Walker, D., & Myrick, F. (2006). Grounded theory: An exploration of process and procedure. *Qualitative Health Research, 16*(4), 547-559.

Wallace, C. S., & Kang, N. H. (2004). An investigation of experienced secondary science teachers' beliefs about inquiry: An examination of competing belief sets. *Journal of research in science teaching*, *41*(9), 936-960.

Wilson, M.S., & Cooney, T. (2002). Mathematics teacher change and development: The role of beliefs. In G. Leder, E. Pehkonen, G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 149-160). Netherlands: Kluwer.

# STATUS OF PRE-SERVICE TEACHERS' UNDERSTANDING OF PROBABILITY AND STATISTICS

Tracey Gorham-Blanco
Colorado State University Pueblo

Scott A. Chamberlin
University of Wyoming

*In this paper, data from the Statistical Reasoning Assessment (Garfield, 2003) taken by pre-service teachers (PST), n=134, in the intermountain region (United States) is detailed. The manner in which PST correctly reasoned and the misconceptions that they held is discussed. PST have moderate correct reasoning skills (M=.49, SD=.1) and a moderate frequency of misconceptions (M=.28, SD=.07). Areas of promise are PST's ability to distinguish between correlation and causation and to interpret probability data, as well as to realize the importance of large samples. Sampling variability, selection of an appropriate average, and the ability to compute probabilities were identified as deficits.*

## Introduction

The purpose of this paper is to explicate data secured in fall 2015, during the final week(s) of PST method's semester. In this paper, specifics of concepts in statistics and probability are provided in an attempt to help teacher preparation programs consider prospective changes to their content areas. PST in elementary education were utilized to identify strong and weak understandings of statistics and probability.

## Objectives of the study

The objective of the study was to determine concrete evidence of PSTs' content knowledge in statistics and probability to identify strong and weak conceptual understanding. In so doing, teacher preparation programs might consider such evidence in relation to existing data.

## Literature Review

The literature in statistics education, which subsumes probability education as well, is not long. In fact, in 1992, Shaughnessy showed that literature in statistics education was quite sparse, with approximately 150 publications in the previous 30 years. In a follow-up study published in 2007, he identified more than 300 such publications. Statistically speaking, this is a fourfold increase (twice as many publications in half the time). This finding led him to claim that no other area in mathematics education was growing quite so quickly, but he alluded to the fact that this growth may likely have been a result of such low number of publications previously. The very notion that over 300 publications were identified enabled him to make the recommendation that the two intricately intertwined fields, should be separated and referred to as statistics education and probability education. The latter recommendation (i.e., for probability education), never really seemed to be accepted by mathematics educators. Of the

300 publications from 1992 to 2007, at least half of those pertained to statistics (and probability) education in schools (Shaughnessy, 2007). Near the time of the robust propagation of research in statistics education, two journals dedicated solely to the field, the *Journal of Statistics Education* and the *Statistics Education Research Journal* were created. Still, the ability to assess students' and teachers' understanding of concepts in statistics and probability was notably absent. The ability to assess students' understanding of concepts was bolstered with international assessments such as PISA (Programme for International Student Assessment), TIMSS (Trends in International Mathematics and Science Study), and most specifically the NAEP or National Assessment of Educational Progress (Gorham-Blanco, 2016).

However, in the late 1990s, Garfield worked with several colleagues to create a psychometrically stable instrument, designed to assess teachers' understanding of such concepts. By 2002, she had developed and disseminated the *Statistical Reasoning Assessment* (Garfield, 2003), which enabled teacher educators the opportunity to critically assess what teachers knew, or did not know, about statistics and probability concepts. This appears to be the most widely available and therefore widely used instrument for such purposes. Perhaps the only study of note in which it was not used was Dollard's (2011) dissertation. In this study, he documented how pre-service teachers reasoned about standard (e.g., a die) and non-standard shapes (Monopoly ® hotels) as they were rolled.

Garfield's work was not only contributory in enabling teacher preparation experts the opportunity to assess content knowledge in statistics and probability, but it helped define what constitutes the field of (concepts in) statistics and probability. Though experts are not in full agreement, statistical reasoning is typically regarded as comprised of several concepts, many of which were outlined by Garfield (2003) in her instrument. The Guidelines for Assessment and Instruction in Statistics Education, or GAISE, (Franklin, Kader, Mewborn, Moreno, Peck, Perry, & Scheaffer, 2007) also helped determine the concepts that comprise statistics education. A comprehensive, but perhaps not exhaustive list, may be comprised of:

- Probability, means of center and outliers, ratio, combinatorics, independence, sampling (and variability), correlation and causation,
- Group comparison, data interpretation, law of large and small numbers, representativeness, equiprobability

Given a baseline understanding of the concepts investigated in this study, the methods section is important to understand the manner in which the study was conducted.

## Method

Participants, *n*=134, were comprised of PST seeking either an initial bachelor's degree and within five years of high school graduation (so-defined as traditional students) or individuals past five years of high school and/or seeking a second bachelor's degree or master's degree in teacher education. The sample was taken from two universities in the intermountain region, along the Front Range in the Rocky Mountains. This quantitative study enabled researchers to identify correct reasoning skills and misconceptions among the PST. The 20 item *Statistical Reasoning Assessment* (Garfield, 2003) was used to collect data given its solid psychometric properties with a test-retest reliability of 0.7 for the correct reasoning portion of the instrument and reliability of 0.75 for the misconception portion. Experts in the field of statistics education verified face validity for the instrument.

To collect data, the second author visited seven college classrooms during the last week of methods classes in fall 2015. Methods classes are defined as those courses taken prior to student teaching in which PST learn how to teach their respective curricula and content. Demographic data were also collected, but all of it is not used in this paper, given the voluminous nature of the data. Data were analyzed using descriptive and inferential procedures. In total, data were analyzed with consideration for eight correct reasoning concepts and eight misconception concepts. Each type of reasoning was analyzed with one of four pieces of demographic categories in mind: gender (man/woman), degree sought (first time bachelor's or post-baccalaureate/master's), student status (traditional/non-traditional), and previous coursework in statistics (yes/no). To clarify demographic definitions, a first time bachelor's degree seeking student had never attained a bachelor's degree. This, in opposition to counterparts that had attained a bachelor's degree and were seeking teacher certification at the elementary level through a second bachelor's degree (so-called a post-baccalaureate degree) or a master's degree. A traditional student was defined as one within five years of high school graduation. Others, those past five years, were considered non-traditional students. Previous coursework was defined as any course in statistics and/or probability outside of those taken in the general teacher preparation program. The no previous coursework descriptor was reserved that had only taken coursework in the typical teacher preparation program (which did include a course in which concepts in algebra and statistics and probability were covered). As mentioned previously, all of the data cannot be elucidated in this paper; only data in which significant differences exist is discussed. For the full version of the study, including all data analysis and commentary, see Gorham Blanco (2016).

Table 1

| | Gender* | Degree Sought | Student status | Previous coursework |
|---|---|---|---|---|
| CC2: Selecting an appropriate average | M>F | | | |
| CC5: Sampling variability | | UND>PBM | | |
| CC6: Correlation versus causation | | | | PC>NPC |
| CC8: Understands importance of large samples | | UND<PBM | NT>TR | |
| Overall correct reasoning score | | UND<PBM | | |
| MC1: Average misconception | M<F | | | |
| MC2: Outcome orientation misconception | M<F | UND>PBM | NT>TR | |
| Overall misconception score | | UND>PBM | NT<TR | |

Key
M=Male, F=Female
UND=Undergraduate students, PBM=Post baccalaureate & Master's students
NT=Non-traditional student, TR=Traditional student
PC=Previous coursework, NPC=No previous coursework

*It is important to note that the number of men (18) and women (116) was quite imbalanced so the difference in the two categories may be problematic to ascertain.

**Furthermore, a high score on CC (correct conception) is desirable and a high score on MC (misconception) is undesirable.

Table 2

| Correct reasoning skills | Items and alternatives |
|---|---|
| CC2: Understands how to select an appropriate average | 1d, 4ab, 17c |
| CC5: Understands sampling variability | 14b, 15d |
| CC6: Distinguishes between correlation and causation | 16c |
| CC8: Understands importance of large samples | 6b, 12b |

| Misconceptions | Items and alternatives |
|---|---|
| MC1: Misconceptions involving averages | |
| a) Averages are the most common number | 1a, 17e |
| b) Fails to take outliers into consideration when computing the mean | 1c |
| c) Compares groups based on their averages | 15b, f |
| d) Confuses the mean with the median | 17a |
| MC2: Outcome orientation misconception | 2e, 3a, b, 11a, b, d, 12c, 13b |

## Results and discussion

With this data, several significant differences were identified at the 0.05 alpha level and these differences are listed below in table 1. In table 2, respective items from the Statistical Reasoning Assessment are presented. Listing the actual selected response stems and items would require excessive space, not afforded in this publication. For a detailed discussion of the instrument, see Garfield (2003).

## Implications

At least three implications come from this study. First, the level of conceptual understanding, including misconceptions, in statistics and probability needs addressed with elementary PST. This implication arose as a result of the performance of the PST on this instrument. That is to say, the overall level of conceptual understanding, 49% (which should ideally be a high value) and the overall level of misconceptions, 28% (which should ideally be a low value) appeared to be respectively lower and greater than what would be considered acceptable for teachers beginning their career. Second, it is likely that such a study should be conducted with in-service elementary teachers to gain an appreciation for their level of conceptual understanding of seminal concepts in statistics and probability. Third, given this report and other anecdotal information that substantiates such claims, it is likely that the domain of statistics and probability will most assuredly continue to be underrepresented as a mathematics domain as long as conceptual understanding is low and misconceptions are high. Hence, efforts need to be invested in order to raise the level of conceptual understanding of PST and to lower the level of misconceptions among PST in elementary grades.

## Limitations

Throughout the data collection process, the two researchers periodically contemplated the level of conceptual understanding of secondary PST and in-service teachers. It is an easy assumption to make that if individuals have completed coursework and are ostensibly prepared to lead students in grades 6-12, then they must be content rich. This assumption cannot be made. Hence, the findings can only be applied to elementary PST and not secondary PST. More appropriately, it may be problematic to apply the findings to the overall population of elementary PST. This is because the sample used was relatively small, fairly homogeneous, and therefore not particularly representative of all elementary PST. Though an instrument with solid psychometric properties was used, the demographic nature of the participants may limit the overall findings and their generalizability.

## Areas for future research

To address the shortcomings of the findings, it would be necessary to conduct a larger, more longitudinal study with a larger *n* and/or with a mixed methods approach in an attempt to attain greater generalizability. In considering the limitation that the findings do not apply to all teachers, a parallel investigation with secondary PST (and possibly elementary) in-service teachers is necessary. This is an area in need of investigation, as it may be the case that despite more rigorous coursework in the general field of mathematics for secondary teachers, they may not have adequate understanding of concepts in statistics and probability for various reasons. The first reason may be that a statistics and probability course was not required of them. The second reason may be a result of such content courses not being conceptually based, as is needed with teachers.

## References

Dollard, C. (2011). Pre-service elementary teachers and the fundamentals of probability. *Statistics Education Research Journal, 10*, 27-47.

Franklin, C., Kader, G., Mewborn, D., Moreno, J. Peck, R., Perry, M., & Scheaffer, R. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-K – 12 curriculum framework.* Alexandria, VA: American Statistical Association.

Garfield, J. (2003). Assessing statistical reasoning. *Statistics Education Research Journal, 2,* 22-32.

Gorham-Blanco, T. (2016). *Statistics Education of Elementary Teachers: Pre-service Teachers' Statistical Reasoning and Misconceptions.* Dissertation submitted to the University of Wyoming.

Shaughnessy, J. M. (2007). Research on statistics learning and reasoning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 957-1009). Reston, VA: National Council of Teachers of Mathematics.

Shaughnessy, J. M. (1992). Research in probability and statistics: Reflections and directions. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-494). Reston, VA: National Council of Teachers of Mathematics.

# INVESTIGATING PRE-SERVICE TEACHERS' PREFERENCE AND AWARENESS ON AN ONLINE MATHEMATICS METHODS COURSE

Hsing-Wen Hu
University of Alaska Anchorage
hhu2@alaska.edu

Grant Sasse
University of Alaska Anchorage
gmsasse@alaska.edu

*Teaching a mathematics methods course online has become more in demand in current teacher education. However, there are few studies to investigate how online teaching impacts pre-service teachers' (PSTs) preference and awareness toward this implementation. This study focused on discovering PSTs' perspectives before they completed the online mathematics course. It revealed that PSTs preferred to take face-to-face courses instead of online courses. They also acknowledged the advantages and disadvantages from both teaching modes, respectively. Finally, this study provides some implications for mathematics educators to consider when they design the curriculum for PSTs.*

## Introduction

There have been many studies about how to carry out education courses through online methods (Kim, Park, & Cozart, 2014; Zaranis, Kalogiannakis, & Papadakis, 2013). These studies have been successfully reforming teacher preparation on what technological tools and strategies can be used in teaching and learning settings. In addition, developing PSTs' technology literacy, especially on mathematics education, has been discussed in the past decade (Zaranis et al., 2013). However, few studies have focused on what preference and awareness PSTs may have toward integrating technology into their mathematics instruction. This study represents an effort to investigate what awareness and preferences the pre-service teachers had toward online teaching before they took the mathematics methods course online.

## Objectives of the study

This research is part of a larger study investigating PSTs' awareness and perspectives on an online mathematics methods course. As we know, teaching courses through online or distance modes has become in high demand for teacher education. However, the current practices may neglect PSTs' awareness and perspectives toward online teaching. This may negatively influence their beliefs and capacities on transferring what they learned from an online mathematics course to their curriculum design and teaching performance. Thus, for this study, we were interested in discovering the phenomena in depth by comparing and analyzing qualitative descriptions related to online teaching preference and awareness from PSTs who were enrolled to take a distance mathematics methods course. Our research questions included: (a) Do pre-service teachers prefer to take mathematics methods courses online?

Why? (b) What is the pre-service teachers' awareness toward online mathematics methods before they took the course?

## Related Studies

Many studies revealed that online teaching may amplify teachers' teaching capacity (Baran & Cagiltay, 2006; Kinney & Robertson, 2003) which can be one of many reasons why online teaching continues to become more popular in current mathematics education. Surrette and Johnson (2015) examined several studies about professional development delivered online and found that teachers were able to demonstrate the following characteristics and skills after the workshop: (a) clarify and deepen their understanding of subject-specific knowledge, (b) transform and apply an innovative subject-specific pedagogical approach in their classroom, and (c) deliver instruction to students that improved their academic achievement in the subject area. In another study, Yang and Liu (2004) found that the online environment enabled students to clarify their content-pedagogical knowledge toward mathematics, develop instructions of conceptual understanding in mathematics classroom, and it provided them opportunities to practice new skills and apply adequate resources.

However, pre-service teachers' resistance or destructive perspectives toward online teaching becomes a challenge for mathematics educators applying it in the educational setting (Redmond, 2011). This issue is driven from their negative experiences with online education in the past. First of all, learners who experienced a disorganized online teaching curriculum design were very dissatisfied and became confrontational when having to take an online course again (Lee, 2014). Second, unnecessary online activities drew negative perspectives (Huss & Eastep, 2013). Third, online teaching conducted by some instructors were limited by one way communication (Ku, Akarariworn, Rice, Glassmeyer, & Mendoza, 2011). Fourth, PSTs lacked the necessary technological knowledge and skills to thrive in the online course (Choy, NcNicke, & Clayton, 2002). Although these PSTs were raised in technological ages, they encountered hardware and software issues when they were taking the online courses (Ku et al., 2011). All of the factors may cause negative preference and awareness toward online teaching.

The dispositional preference and awareness toward online teaching refer to students' thoughts about online education and what they believe to be the necessary components for their success in this online environment (Huss & Eastep, 2013). Learners' positive reference and awareness have been verified as one of the major elements in determining the quality of online education (Callie, Balcikanli, Calli, Cebeci, & Seymen, 2013). In addition, Bolliger and Halupa (2012) examined learners' anxiety toward online education and the impact this had on their perspectives. The study found a significant negative correlation between anxiety and the learners' preference and awareness.

In order to develop learners' positive awareness and perspectives toward online teaching, several points need to be considered when the instructors design their curriculum. First, strengthen pre-service teachers' belief on their capacities to handle online courses which may help them to achieve the outcomes in the online educational environment (Liaw, 2008). Second, the online instructors' own attitudes and supporting capacity can make a dynamic difference on students' preferences and awareness towards the online teaching. Third, social presence in online courses also plays a crucial role on students' preferences and awareness (Lowenthal & Dunlap, 2011). In addition to the previous factors, Sun, Tasi, Finger, Chen, and Yeh (2008) identified seven critical factors that impacted students' preference and awareness toward online courses. They include instructor attitude, computer anxiety, course flexibility, perceived usefulness, course quality, perceived ease of use, and diversity of assessment.

## Context of the study

In order to examine PSTs' awareness and perspective toward an online mathematics methods course, a large study was conducted. This study first utilized a pre-reflection which focused on investigating PSTs' preference and awareness before they took the online mathematics methods course. In this study the ASSURE model (Smaldino, Lowther, & Russell, 2012) was utilized— a model that leads educators to plan systematically for effective use of technology and media — as a tool to conduct distance teaching in a mathematics method course. The curriculum design for the online mathematics methods course includes four categories: investigating adequate implementation content, creating an effective distance teaching environment, establishing distance learning assessments, and conducting mutual and efficient communication. After the participants completed the online mathematics methods course, the PSTs wrote a post-reflection which was used as the main source for examining if PSTs' awareness and preferences changed as a result from taking the online mathematics methods course.

## Method

### The Participant

This study had 13 participants from a northwest university. In order to include and accommodate students who live in remote places, the education program adopted an alternative system to offer a mathematics methods course. In other words, each fall semester this education program offers a distance online mathematics methods course and each spring semester offers this course face-to-face. Pre-service teachers who took the distance online mathematics methods course were selected to participate in this study. All participants had previously completed one educational technology course and at least two math content courses before they were allowed to take the mathematics methods course.

### Data collection

In order to investigate PSTs already held awareness and preferences towards taking an online mathematics methods course, the principal investigator collected participant pre-reflections at the beginning of the course. In this pre-reflection, participants addressed the following questions: Describe your experiences in face-to-face and distance courses. How were they similar? How were they different? What did you like about each? What did you not like about them? Why did you choose to take this math methods course online? What advantages or disadvantages do you anticipate having while taking the math methods course online? How do you think this format (the online math methods course) will impact your ability to teach elementary students face-to-face?

### Data analysis

Analysis of the data followed the constant comparative method (Lincoln & Guba, 1985). The raw materials were read and reread by the researchers. The researchers independently noted emergent categories, then compared the categories and developed agreement for the possible themes. Based on the agreement, the researchers reread and coded the data. All coded data was read by another person to verify the accuracy of coding. Once a consensus was reached, the researchers identified patterns that emerged within and across the data.

## Results and Discussion

The analysis of pre-reflection data revealed that PSTs' preference and awareness of online course delivering is similar to what is reflected in the literature, but they also contribute some insights important for curriculum design in mathematics education. Although participants were enrolled to take an online mathematics methods course, six participants (47%) indicated intent to take the course face-to-face if it was available because they believed a face-to-face course to be more personal, interactive, comprehensive, and reliable. There was one participant (7%) wanting to take the online mathematics methods course because it was more convenient. Interestingly, two participants (15%) liked both course delivering methods and this was based on their positive experiences in the past. Four participants (31%) did not express their preference in their pre-reflections (See Table 1).

Table 1.
*Pre-service teachers' preference on the course delivery method before taking an online mathematics methods course*

| Preference | N (%) | Reasons |
|---|---|---|
| Preferring to distance teaching (DI) | 1 (7%) | o F2F class not easy fit into work schedule.<br>o Not enjoy city and being around crowds. |
| Preferring to face to face teaching (F2F) | 6 (47%) | o Fitting various learning styles<br>o Having social interactions<br>o Course being more comprehensible for participants<br>o Directly seeing experienced instructor's teaching styles<br>o More relaxing |
| Preferring both DI & F2F | 2 (15%) | o Having good experiences with both teaching modes<br>o Both modes having its own advantages |
| No reference | 4 (31%) | N/A |

Relating to participants' preference, this study explored PSTs' pre-service awareness/perspective on course delivering modes, especially related to math teaching. The data showed that participants viewed distance teaching and face-to-face teaching as having its own advantages and disadvantages, respectively (see Table 2), and this was consistent to previous studies. One of this study's research interests of how the course delivery method may impact PSTs' math teaching was not conclusive due to the data not providing sufficient information (see Table 2). Participants simply addressed belief/motivation and teaching strategies briefly and indirectly. There was no further discussion from participants on their beliefs regarding how an instructor structures a course, the nature of mathematics, or the technological modes/tools that impact PSTs' ability to teach elementary math lessons.

**Implications**

The goal of this study was to discover what preferences and awareness participants had of taking an online mathematics methods course, before they actually completed the distance course. The purpose of this data was intended to help the researchers adjust the online mathematics method course, based on the participants' perspectives. It seems that the online delivery of courses are gradually being accepted as a norm or an acceptable standard by current higher education students, especially by students higher education may deem nontraditional students (Hodges & Cowan, 2012). Although this may be, according to this study, PSTs are still hesitant to take an online mathematics methods course because the course might lack interaction, visual models, and clear communication, all of which are important to the education profession. As a result, there are many new technology teaching systems and tools being developed to address these concern (Smaldino et al., 2012). Therefore, the mathematics educator should integrate these new technological systems and tools into the design of their online course, so PSTs can learn mathematics pedagogical knowledge at the same level of quality as if they were learning in a face-to-face course. Furthermore, educators will amplify their mathematics teaching because they are familiar with these technological systems and tools. Future studies can focus on how best to integrate new technological systems and tools into the classroom, in order to best educate PSTs.

Table 2.

*Pre-service teachers' awareness/perspectives on the modes of course delivering before taking online mathematics methods course*

| Categories & themes | | Awareness/perspectives toward DI | Awareness/perspectives toward F2F |
|---|---|---|---|
| General conceptions | Advantages | o Availability, convenience<br>o Self-guided learning<br>o More flexibility<br>o Learn to integrate technology into teaching<br>o Reaching to all students | o Build relationship/peer support<br>o Understand assignment<br>o Can model professor easier<br>o Easier to ask questions<br>o Good for visual learners |
| | Disadvantages | o No physical interaction<br>o Less social<br>o No non-verbal communication<br>o Understand assignment less<br>o Technology frustration | o Travel an parking<br>o More time commitment and less time in practicum setting |
| Impact on math teaching | Belief/Motivation | o Won't impact PST's teaching abilities<br>o Interactive & involved instructions make lesson successful<br>o Won't have different with F2F if having excellent support.<br>o Being fitted self-motivated learners | o Can promote community and connection<br>o Well prepare PSTs<br>o All Ed courses should be F2F |
| | Teaching strategies | o Present information with PowerPoint<br>o Assignment/content in Blackboard system | o Present information with PowerPoint<br>o Assignment/content in Blackboard system |

## References


Baran, B., & Cagiltay, K. (2006). Teachers' experiences in online professional development environment. *Turkish Online Journal of Distance Education*, *7*(4), 110-122.

Bolliger, D., & Halupa, C. (2012). Student perceptions of satisfaction and anxiety in an online doctoral program. *Distance Education*, *33*, 81-98.

Calli, L., Balcikanli, C., Calli, F., Cebeci, H., & Seymen, O. (2013). Identifying factors that contribute to the satisfaction of students in e-learning. *Turkish Online Journal of Distance Education*, *14*, 85-101.

Choy, S. C., McNickle, C., & Clayton, B. (2002). Learner expectations and experiences. Student views of support in online learning. In H. Guthrie (Ed.), *Online learning: Research readings* (pp. 106-122.) Retrieved from http://eprints.qut.edu.au/7250/1/7250.pdf

Hodges, C. B., & Cowan, S. F. (2012). Preservice teachers' views of instructor presence in online courses. *Journal of Digital Learning in Teacher Education*, *28*, 139-145.

Huss, J. A., & Eastep, S. (2013). The perceptions of students toward online learning at a Midwestern university: What are students telling us and what are we doing about it? *Inquiry in Education*, *4*(2), 1-20.

Kim, C. M., Park, S. W., & Cozart, J. (2014). Affective and motivational factors of learning in online mathematics courses. *British Journal of Educational Technology*, *45*(1), 171-185.

Kinney, D. P., & Robertson, D. F. (2003). Technology makes possible new models for delivering developmental mathematics instruction. *Mathematics and Computer Education*, *37*(3), 315-328.

Ku, H. Y., Akarariworn, C., Rice, L. A., Glassmeyer, D. M., & Mendoza, B. (2011). Teaching an online graduate mathematics education course for in-service mathematics teachers. *The Quarterly Review of Distance Education*, *12*(2), 135–147.

Lee, J. (2014). An exploratory study of effective online learning: Assessing satisfaction levels of graduate students of mathematics education associated with human and design factors of an online course. *The International Review of Research in Open and Distance Learning*, *15*(1), 111-132.

Liaw, S. (2008). Investigating students' perceived satisfaction behavioral intention, and effectiveness of e-learning: A case study of the Blackboard system. *Computers and Education*, 51, 864-873.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Lowenthal, P. R., & Dunlap, J. (2011). *Investigating students' perceptions of various instructional strategies to establish social presence*. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Redmond, P. (2011) *From face-to-face teaching to online teaching: pedagogical transitions.* In: ASCILITE 2011: 28th Annual Conference of the Australasian Society for Computers in Learning in Tertiary Education: Changing Demands, Changing Directions, 4-7 Dec 2011, Hobart, Australia.

Smaldino, S.E., Lowther, D.L., & Russell, J.D. (2012). Instructional technology and media for learning (10th ed.). Upper Saddle River, NJ: Prentice Hall.

Sun, P. C., Tsai, R. J., Finger, G., Chen, Y.Y., & Yeh, D. (2008). What drives a successful elearning? An empirical investigation of the critical factors influencing learner satisfaction. *Computers and Education*, *50*, 1183-1202.

Surrette, T. N., Johnson, C. C. (2015). Assessing the ability of an online environment to facilitate the critical features of teacher professional development. *School Science and Mathematics*, *115*(6), 260-270.

Yang, S. C., & Liu, S. F. (2004). Case study of online workshop for the professional of teachers. *Computers in Human Behavior*, *20*(6), 733-761.

Zaranis, N., Kalogiannakis, M., & Papadakis, S. (2013). Using mobile devices for teaching realistic mathematics in kindergarten education. *Creative Education*, *4*(7A1), 1-10.

# SUPERVISION IN THE CONTENT AREAS: CASE STUDIES FOR DEVELOPING INSTRUCTIONAL LEADERS

Sarah Quebec Fuentes
Texas Christian University
s.quebec.fuentes@tcu.edu

Mark A. Bloom
Dallas Baptist University
markb@dbu.edu

Daniel R. Ilaria
West Chester University
dilaria@wcupa.edu

Jo Beth Jimerson
Texas Christian University
j.jimerson@tcu.edu

*Even well-prepared school leaders find themselves supervising teachers in grades and/or content areas in which they lack experience. To fill such gaps, this paper explores the use of case studies in elementary, middle, and high school mathematics and science instruction to support the development of K-12 instructional leaders. The cases incorporate preliminary activities, narratives, discussion questions, suggested activities, supplementary resources, and case facilitation notes. Intentional use of these cases will expose developing school leaders to the nuances that distinguish mathematics and science instruction from other content areas and better prepare leaders to support teachers in these subjects.*

## Introduction

School leaders are essential to the K-12 environment. One of the critical roles of school leaders is to develop and support effective instruction; that is, school leaders should be instructional leaders.

> The main underlying assumption [of instructional leadership] is that instruction will improve if leaders provide detailed feedback to teachers, including suggestions for change. It follows that leaders must have the time, the knowledge, and the consultative skills needed to provide teachers – in all the relevant grade levels and subject areas – with valid, useful advice about their instructional practices. (Louis et al., 2010, p. 11)

Instructional leadership is challenging. First, school leaders have a multitude of competing responsibilities including operating with various stakeholders, managing resources (financial, material, and personnel), defining and implementing short- and long-term goals, providing a safe environment, and focusing on classroom practice (National Policy Board for Educational Administration, 2015). Second, school leaders are most likely specialists in a particular grade band or content area. Therefore, school leaders must enhance their knowledge beyond their area of expertise in order to provide constructive feedback to all teachers related to pedagogy *and* content.

## Purpose of the Paper

The purpose of the present paper is to share an approach to support the development of instructional leaders in the subject areas of science and mathematics. Specifically, through the case method, school leaders engage with case studies, which focus on issues of instructional leadership (rather than managerial concerns), in interactive and in-depth ways through the lens of the responsibilities of instructional leaders. In what follows, the paper includes the instructional framework grounding the case study approach; a description of the various components of each case; synopses of three mathematics and three science case studies one each in an elementary, middle, and high school setting; and implications for the roles of science and mathematics teacher educators (TEs) in the growth of instructional leaders.

## Instructional Framework

The approach for developing instructional leaders described herein is guided by the analogy of *postholing* in combination with the case method. To install a fence on a farm or ranch, one of the first steps is digging holes for the posts. The postholes need to be deep but not very wide in order to support the fence. Similarly, school leaders cannot be expected to have expertise in all content areas and/or grade levels within a school. However, they can have a depth of knowledge about some aspects of the various content areas, what Stein and Nelson (2003) term leadership content knowledge:

> All administrators have solid mastery of at least one subject (and the learning and teaching of it) and … they develop expertise in other subjects by "postholing," that is, conducting in-depth explorations of an important but bounded slice of the subject, how it is learned, and how it is taught. (p. 423)

By postholing, school leaders are able to enhance their pedagogical content knowledge (Shulman, 1986) across the curriculum.

Case studies have the potential to support school leaders in the process of postholing. The case method has a long history in fields such as business, law, and medicine (Merseth, 1991), and Shulman (1986) argues for the use of cases in education. They portray the practical and theoretical facets of instruction. In implementing cases, a reflexive relationship between the case itself and discussions about the case exists. When discussions focus on the critical content of the case, they have the potential to foster problem solving, reflection, and intentional action related to issue-laden scenarios; encourage the pursuit of personal development; and cultivate collaboration among colleagues (Merseth, 1991). The present paper describes the use of the case method to expose developing instructional leaders to the nuances of mathematics and science instruction.

## Practice or Innovation

The cases can be used with preservice leaders in educational leadership (EL) programs and inservice leaders in professional development (PD) programs. In addition to a problems-based case narrative, each case includes pre- (introductory information, preliminary activities, and focus areas) and post-case (discussion questions, suggested activities, and suggested resources) information and activities. For those leading these activities, facilitation notes elaborate upon the content and pedagogical content foci of the case and link them to the practices presented in the case.

Prior to the case, school leaders read a brief preview of the instructional emphases of the case. By engaging with the preliminary activities (e.g., completing readings, watching videos, and interacting with exemplar teachers), school leaders start to build their content and pedagogical content knowledge. Lastly, the focus areas remind school leaders of aspects of the case to which to pay particular attention. After reading the case, school leaders participate in discussions centered on questions related to the content area, the teacher's practice, and the role of the instructional leader in the case and two to three activities (including at least one role play) designed to extend and connect learning related to the case. Suggested resources (e.g., organizations, websites, and further readings) provide school leaders with tools to continue their professional growth. The pre- and post-case activities are crucial for learning through the case and connecting theory and practice.

## Classroom Examples

### Science Cases

**Elementary.** The elementary science case engages developing school leaders in understanding and evaluating inquiry-based science instruction (IBI). IBI can take many forms depending on the educational goal for the students, ranging from "highly structured by the teacher" (as when guiding students to particular learning objectives) to "free-ranging explorations" (more similar to the actual process of science) (NRC, 2000, p.10). Students can test predictions and revise ideas as needed; allowing students to make mistakes and correct for them is a critical part of IBI. School leaders must understand the messiness of IBI (and how the instructional approach supports student learning) before they evaluate this form of teaching.

In this case, Ms. Fox, a STEM Lead Teacher, has been hired by Principal Weston to ensure that teachers at all grades develop the skills to integrate IBI into their instruction. Ms. Fox is working with second grade teacher, Mr. Jackson (a first-year teacher who is new to inquiry-based instruction) with the support of experienced fourth grade teacher Mrs. Hernandez. Principal Weston participates in the planning of the lesson, observes its implementation, and offers his own perspective of the experience in a post-lesson debrief with all three teachers. The case demonstrates the collaborative nature of supporting teachers who are struggling with IBI and the necessary knowledge base for school leaders to foster this type of instruction.

School leaders should be prepared to see a learning environment that, at first glance, might look quite unorthodox. They must watch for several key elements of IBI, which are outlined in the 5-E model of instruction: engage, explore, explain, elaborate, and evaluate (Bybee, 1997). Each of the five Es of IBI should be evident in the lesson. To begin, school leaders should watch for the *hook* that makes the lesson exciting and relevant. Second, they should observe students actively working with materials (e.g., physical materials, online resources, or books) to help students develop concepts and enact the process skills of science. School leaders should hear students describing, in their own words, what they have learned and applying the newly acquired knowledge to other contexts. Last, school leaders must recognize authentic assessment that ensures all students have mastered the learning objectives. School leaders must understand these critical aspects of IBI that are unique to science instruction.

**Middle school**. Science content can, at times, cause discomfort for students and their families due to political, economic, or religious implications (Authors, 2014; Author, 2007). The middle school science case exposes developing school leaders to instruction about a religiously sensitive subject matter, evolutionary theory, and how integrating nature of science (NOS) into instruction can mitigate the tension that some students might feel.

In this case, first year teacher, Mr. Guerra, is worried about creating conflict among his students when teaching evolution and natural selection. He receives support from senior teacher, Mrs. Temple, who guides him in incorporating NOS into his instruction to help students distinguish between scientific and non-scientific explanations. Mr. Guerra follows Mrs. Temple's advice and pre-frames his instruction with two NOS activities (Lederman & Abd-El-Khalick, 1998; Lederman, Gnanakkan, Bartels, & Lederman, 2015). Through these activities, Mr. Guerra reinforces that empirical evidence must be available to support scientific claims and that science is limited to utilizing natural processes to explain phenomena. This case highlights the importance of incorporating NOS instruction throughout the science curriculum.

Guiding science education reform documents have long emphasized the importance of interweaving NOS throughout science instruction (AAAS, 1993; NRC, 1996; NGSS Lead States, 2013). The case starts developing school leaders on the path of learning the tenets of NOS beyond those emphasized (i.e., tentative, subjective, creative, social/cultural, and observations/inferences). With an understanding of these basic elements of NOS, school leaders are able to justify what is included or excluded from a science curriculum and determine if science lessons are infused with NOS both directly and indirectly. When observing science lessons, school leaders should look for evidence of NOS instruction so that not just the findings of science are emphasized, but also how scientific investigations are conducted, the types of questions that science can and cannot answer, and the distinctions between scientific and non-scientific knowledge claims.

**High school.** Science instruction can be implemented through myriad pedagogical approaches including IBI, problem-based learning, or hands-on laboratory investigations. Teachers must be sure to choose an appropriate strategy to match the learning objectives. School leaders need to be prepared to support their teachers when employing novel approaches.

In this case, high-school science teacher, Rob Joyce, decides to employ a new form of instruction, flipping the classroom, when he is teaching climate change, a highly complex and misconception-laden subject. Mr. Joyce provides the students with YouTube lectures prior to class and then uses class time to support students in investigating and refuting common misconceptions about global warming. To showcase this approach, Mr. Joyce has invited his Principal, Mrs. Nguyen, to observe instruction. Upon enacting the lesson, however, Mr. Joyce learns that uprooting scientific misconceptions might be more difficult than he predicted. Some students actually reinforced misconceptions rather than countered them. School leaders need to recognize the “unnatural nature of science” (Wolpert, 1992) and the challenges this poses for teachers.

School leaders must be prepared to observe many different types of teaching and learning that occur in a science classroom. Because of the complex nature of some science content, they should also be prepared to see students struggling with the subject matter. Instead of focusing on students’ immediate success, school leaders should watch for ways in which teachers support students in accommodating challenging and counterintuitive science ideas.

## Mathematics Cases

**Elementary.** Providing instructional feedback to mathematics teachers can be challenging when school leaders are not content area experts. The elementary mathematics case engages school leaders in understanding effective research-based teaching practices and in developing an understanding about teaching fractions. Two of the mathematics teaching practices from *Principles to Actions* (NCTM, 2014) are highlighted in the case as well as the use of number lines to teach fractions.

In this case, a novice teacher, Ms. Adams, experiences difficulty identifying and remediating a student misconception with respect to comparing fractions. She then works with the mathematics coach, Ms. Schratz, to improve her instruction. In a subsequent lesson observed by the principal, Ms. Adams engages students in mathematical practices that *build procedural fluency from conceptual understanding* and *support productive struggle in learning mathematics* (NCTM, 2014). Despite the success of the collaboration between Ms. Schratz and Ms. Adams, the principal's observation feedback revealed concerns about how the lesson was delivered.

School leaders must be open to alternative methods of teaching and learning mathematics so all students can experience success. While school leaders might be familiar with students completing multiple practice exercises, they must realize that such procedural fluency does not ensure conceptual understanding. Teachers need to provide purposefully designed tasks that require students to productively struggle. School leaders must recognize when students are authentically engaging with mathematics in ways that support a deeper understanding.

**Middle School.** Often mathematics is taught in isolation rather than collaboration. The middle school mathematics case examines a team of teachers working together to help students understand a foundational mathematics topic, ratio and proportional thinking, while trying to implement more formative assessment into their lessons.

In this case, seventh and eighth grade teachers collaborate with Dr. Liliginny, Director of Mathematics, to create a lesson about connecting ratios to graphs. Together, they outline the learning progression to assist students in reaching the lesson goals and develop formative assessment questions for each level of the learning progression that push and probe student thinking. One teacher leads the lesson, and the team meets afterwards to debrief with Dr.

Liliginny. The debriefing session focuses on using the formative assessment feedback to guide subsequent instructional decisions.

This case provides an example of a school leader supporting the development of teachers' content and pedagogical content knowledge to connect student learning with instructional decisions based on formative assessment measures. School leaders should be aware of and enhance their understanding of the big ideas that span grade levels, such as ratio and proportional reasoning in the middle grades (Lobato & Ellis, 2010). Further, to develop these ideas, teachers need to elicit student thinking. School leaders must look for evidence of teachers providing opportunities for making student thinking public and using what they hear to guide instructional decisions.

**High School.** Mathematics instruction can take on many different forms. While historically direct instruction has been a prevalent approach, reform documents (e.g., NCTM, 2000) recommend the incorporation of *meaningful mathematical discourse* (NCTM, 2014). The high school mathematics case introduces school leaders to the *5 Practices for Orchestrating Productive Mathematics Discussions* (Smith & Stein, 2011) and how discussion is a powerful tool for supporting student learning.

In this case, an experienced teacher, Ms. Sedgwick, has worked diligently to implement the *5 Practices* in her mathematics classroom. When a new school leader, Mrs. Mitchell, observes part of a lesson, she sees students wrestling with problems as they work in groups. In a post-observation discussion, Mrs. Mitchell questions what the struggling students actually learned. Mrs. Sedgwick describes how she implemented the practices of *selecting*, *sequencing*, and *connecting* to guide the learning process. She purposefully chose groups, in a particular order, to share and identify relationships between the different problem-solving strategies in order to achieve the goals of the lesson.

Teachers employ the *5 Practices* at distinct times: while conceptualizing a lesson (*anticipating)*, during its implementation (*monitoring, selecting*, and *sequencing*), and while sharing and summarizing student learning (*connecting*). If a school leader only observes a portion of a lesson, she may have the impression that no teaching or learning is occurring. School leaders need to understand the role of each of the *5 Practices* and the relationship between them in order to evaluate mathematics instruction involving discussion.

## Implications

Science and mathematics TEs work with preservice and inservice teachers to promote research-based teaching practices. However, the work of TEs may be stymied if school leaders do not know and recognize the purpose of these practices. The present paper describes an approach for developing instructional leaders (preservice and inservice), who lack background in science or mathematics education. This approach has the potential to be especially beneficial in courses offered through EL programs, particularly those focused on curriculum, instruction, and assessment. EL faculty have their own specializations from any particular subject or grade level. This exposes a need for science and mathematics TEs to collaborate with EL faculty to implement the cases. Similarly, the cases can prove highly valuable with inservice school leaders. When conducting PD with inservice teachers, science and mathematics educators can offer parallel PD with school leaders utilizing the cases. With the proposed use of case studies, well-informed instructional leaders can continue the work of science and mathematics TEs to positively impact K-12 teaching.

## References


American Association for the Advancement of Science (AAAS) (1993). *Benchmarks for science literacy*. New York, NY: Oxford Press.

Bybee, R. W. (1997). *Achieving scientific literacy: From purposes to practices*. Portsmouth, NH: Heinemann.

Lederman, N. & Abd-El-Khalick, F. (1998). Avoiding de-natured science: Activities that promote understandings of the nature of science. In W. McComas (Ed.), *The nature of science in science education: Rationales and strategies* (pp. 83-126). Dordrecht, Netherlands: Kluwer.

Lederman, J., Gnanakkan, D., Bartels, S., & Lederman, N. (2015). The e-mail lab. *The Science Teacher (82)*6, 57-61.

Lobato, J., & Ellis, A. B. (2010). *Developing essential understanding of ratios, proportions & proportional reasoning: Grades 6-8*. Reston, VA: NCTM.

Louis, K.S., Leithwood, K., Wahlstrom, K.L., Anderson, S.E., Michlin, M., Mascall, B., … Moore, S. (2010). Learning from leadership: Investigating the links to improved student learning (Final Report of Research Findings). New York, NY: The Wallace Foundation.

Merseth, K. (1991). *The case for cases in teacher education*. Washington, DC: American Association of Colleges of Teacher Education and the American Association of Higher Education.

National Academy of Science (1998). *Teaching about evolution and the nature of science*. Washington, DC: National Academy Press.

National Council of Teachers of Mathematics (NCTM) (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

NCTM (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

National Policy Board for Educational Administration (2015). *Professional standards for educational leaders*. Reston, VA: Author.

National Research Council (NRC) (1996). *National science education standards*. Washington, DC: National Academy Press.

NRC (2000). *Inquiry and the national science education standards: A guide for teaching and learning*. Washington, DC: National Academy Press.

NGSS Lead States (2013). *Next generation science standards: For states, by states*. Washington, DC: The National Academies Press.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Smith, M. S., & Stein, M. K. (2011). *5 practices for orchestrating productive mathematics discussions*. Reston, VA: NCTM.

Stein, M. K., & Nelson, B. S. (2003). Leadership content knowledge. *Educational Evaluation and Policy Analysis, 25*(4), 423-448.

Wolpert, L. (1992). *The unnatural nature of science*. Cambridge, MA: Harvard University Press.

# USING DRAWINGS TO EXPLORE BELIEFS ABOUT DOING MATH

Ben Wescoatt
Valdosta State University
ben.wescoatt@gmail.com

*Students in a mathematics content course for teachers were instructed to create drawings of mathematicians doing math, themselves doing math, and their students doing math. The images were analyzed to explore general beliefs about what it means to do math. Using a modified framework of Farland-Smith (2012), the person, the mathematics, the action, the location, and affect in the drawings were described and compared. While the self-images created appeared to trend toward negative feelings of math, the images created of mathematicians and students were generally positive. Overall, the images adhered to stereotypes, suggesting that the participants held a limited view of doing mathematics.*

## Introduction

Teachers play a key early role in helping students form their perceptions of mathematics and what it means to "do math." A teacher's beliefs can influence the mathematical experiences they have with their students and so can influence the perceptions that the students form (Mewborn & Cross, 2007). If students do not have healthy images of mathematics, they may choose to pursue other vocations, potentially robbing society of valuable mathematical innovation. Thus, exploring pre-service teachers' perceptions related to mathematics is important.

## Objectives of the Study

Specifically, this current study explores perceptions of doing mathematics held by pre-service teachers (PSTs). The study is a continuation of a larger study. While the previous study analyzed the doing of mathematics by mathematicians, this study analyzed images created by the PSTs of themselves doing mathematics and of their current or future students doing mathematics. The objectives of the study were to address the following questions.

1. What are the perceptions that PSTs have of themselves doing mathematics?
2. What are the perceptions that PSTs have of their students doing mathematics?
3. To what extent do PSTs' perceptions of doing mathematics compare when the subject of the image varies among a mathematician, a student, and the PST?

## Theoretical Framework and Related Literature

What it means to do mathematics (or in the colloquial and spirit of the current study, "do math") is a somewhat subjective and philosophical concept. From a survey of twenty-five post-secondary mathematics professors, Latterell and Wilson (2012) formulated a working definition of doing math, stating that in order to be considered doing math, mathematicians must be engaged in creating new mathematics, or as Schoenfeld (1994) stated, "research – what most mathematicians would call doing mathematics – consists of making contributions to the mathematical community's knowledge store" (p. 66). Thus, Latterell and Wilson excluded teachers of mathematics from being considered as mathematicians and only included mathematics professors if they were engaged in research mathematics.

However, not everyone would agree with this view. From an analysis of images drawn by pre-service teachers (PSTs), Wescoatt (2016) hypothesized that PSTs believed teachers of mathematics to be engaged in doing math when they were teaching and explaining math to their students. The results agreed with Chick and Stacey's (2013) redefinition of teachers doing math, that teachers of mathematics act as applied mathematicians in order to solve teaching problems.

The ability to create new mathematics is not necessarily restricted to those individuals well-versed in mathematics. New mathematics can be created as learners construct mathematical knowledge that is new to them (Burton, 2002). An individual mathematics classroom may then serve as a mathematical community. Describing such a classroom experience, Schoenfeld (1994) elaborated, "These students, in their own intellectual community, were doing mathematics. They were, at a level commensurate with their knowledge and abilities, truly engaged in the science of patterns" (p. 67). Thus, students have the ability to reason about mathematical objects and create knowledge new to them and their classmates; that is, students can do math in the same way that mathematicians do math.

This study used participant-made drawings in order to explore the perceptions PSTs have of doing math. The use of drawings to explore concepts has its origins in Goodenough's Draw-a-Man psychological test developed in 1926. The test was adapted through the years, notably as the Draw-a-Scientist test in 1983 by Chambers (Finson, 2002). To analyze scientists at work, Farland-Smith (2012) modified an existing rubric, suggesting analysis along the dimensions of the appearance of the scientist (Appearance), the location in which the scientific activity was taking place (Location), and the scientific activity being conducted (Activity).

Bachman, Berezay, and Tripp (2016) used Farland-Smith's rubric to code participant drawings of themselves doing math; the participants were students in an experimental math/dance class. Previous studies (e.g., Rule & Harrell, 2006; Burton, 2012) have used drawing analysis to explore perception shifts in PSTs enrolled in methods courses, with a focus on affect as merely negative, neutral, or positive. To create a more differentiated affective scale, Bachman, et al., assigned scores to each drawing from 1 to 7. A score of 1 represented

"Extremely Negative," 2 represented "Negative," 3 represented "Math is Unpleasant," 4 represented "Neutral," 5 represented "Math is Pleasant," 6 represented "Positive," and 7 represented "Extremely Positive." For example, to be given a score of 1, the image depicted an extreme act, such as vomiting or intense crying. They then used the scale to explore the effect of an experimental pedagogy on students' perceptions of doing math. Participant-made images of mathematicians and mathematics in other studies have depicted extreme images, some suggesting violence (e.g., Picker & Berry, 2000). These images may have little mathematical content, yet they provide valuable insight into beliefs about math and doing math. Thus, incorporating affect in analysis was sensible.

This study did not attempt to precisely define what "doing math" meant. Rather, it relied on implied meanings from the participant images. However, a broad assumed definition was that doing mathematics is a process of understanding and applying meaning to the world, aligning with the description of mathematics as the science of patterns (e.g., Devlin, 2003).

## Methodology

The study was conducted at a regional university in the southeastern United States. Participants, or PSTs, in the study were undergraduate students in a teacher preparation program. The PSTs were enrolled in one of three sections of a mathematics content course for pre-service teachers. The course was the third in a sequence of four mathematics content courses required by the program. Forty-six PSTs were enrolled in the sections. The PSTs were divided between two disciplines, early childhood education (31, 67.4%) and special education (15, 32.6%). Of these students, 4 (8.7%) were male and 42 (91.3%) were female. Additionally, 2 were Hispanic (4.3%), 10 were African-American (21.7%), and 34 (73.9%) were Caucasian.

During the sixth week of classes, PSTs completed an at-home activity consisting of several drawing activities. Germane to this current study were the following prompts: "Draw a picture of a mathematician doing math," "Draw a picture of yourself doing math," and "Draw a picture of one of your students doing math." The PSTs had approximately one week to create the drawings. The drawings were subsequently collected and scanned to create electronic files. Forty-two PSTs completed the assignment, resulting in 42 sets of drawings to analyze.

As the original drawing prompt requested students to draw a picture of a mathematician, yourself, or one of your students doing math, analysis of the drawings focused on the person, what the person was doing, and what elements in the drawings could be considered mathematical in nature. Thus, the categories of analysis were Action, Mathematics, Appearance, Location, and Affect (Wescoatt, 2016). The Action and Mathematics categories were a splitting of the Activity category from the Farland-Smith framework in order to better capture elements that the participants consider to be mathematical. Elements in each image that would fit in each category were noted and circled with a colored pencil. For example, a purple pencil was used to circle the Pythagorean theorem equation. Then, frequency counts were made of common elements across the images, such as the number of times a whiteboard or chalkboard appeared in the images.

The affective scale and rubric developed by Bachman, et al., were used to score the images. Mean scores within the mathematician, PST, and students groups were calculated. As the images were drawn in a series as part of the same assignment, they were likely not independent. Thus, a non-parametric equivalent of a paired *t*-test, the two-tailed Wilcoxon signed rank test, was utilized to determine whether or not the median change in scores from one participant drawing to the next across all participants was significantly different from zero. If each drawing contained the same level of affect, the change would be zero. To control the familywise error rate, a Bonferroni correction was used to adjust the significance level for determining the rejection of the null hypothesis for each test. Thus, null hypotheses were rejected when $p \leq .05/3$, or .017.

In a follow-up assignment to the drawings, the PSTs shared in the analysis (Mitchell, Theron, Stuart, Smith & Campbell, 2011), reviewing a subset of the drawings of mathematicians doing math. They listed common themes found across the drawings, compared these themes to their individual drawing of them doing math and their student doing math, and compared the drawings to the themes. These comments were used to shape interpretations during image analysis.

**Results and Discussion**

Mathematicians

These images typically showed a person standing in front of a chalkboard or whiteboard, in the act of writing, presenting, or talking about mathematics. The mathematicians appeared to be teachers in a classroom setting. Almost half of the drawings (47.6%) contained either the Pythagorean theorem equation, the energy-mass equivalence ($E = mc^2$), or both. Moreover, 33 drawings (78.6%) included a notion of equivalence such as an equal sign or angle congruency. A doubling sum, such as 1 + 1, occurred in 8 drawings (19.0%); another 6 drawings (14.3%) contained basic operations involving single-digit whole numbers. Symbols such π and ∞ were present in 8 drawings (19.0%). Geometric drawings or concepts, such as angle, were present in 8 drawings (19.0%). A calculator was present in 4 drawings (9.5%). Only one image included more than one person, while one image was an abstract drawing. The other 40 drawings contained only one individual. The mathematician was standing in 33 images (78.6%); to be considered standing, the person's feet needed to be visible. A vertically-positioned rectangle was present in 37 drawings (88.1%), with many of the rectangles drawn to represent a chalkboard or whiteboard. Seven images (16.7%) contained either a table or a desk. Books were not present in any image.

Using a simplified affective scale, images were coded as positive, neutral, negative, or both. That is, the negative subscales (1-3) and the positive subscales (5-7) were compressed into negative and positive. Images coded as both contained positive and negative elements. Twenty-four images depicted the mathematician smiling. Some of the images contained writing on the board such as "I love Math!" Four images depicted the mathematicians in a struggle with math. For example, a man was perspiring in one image and question marks were in a thought bubble of another. These images were coded to be negative (Bachman, et al., 2016; Rule & Harrell, 2006). Overall, 25 images were positive (59.5%), 12 images were neutral (28.6%), 4 images were negative (9.5%), and 1 image contained both positive and negative elements (2.4%).

PSTs

In contrast with the images created of the mathematicians, the images created of the PSTs doing math indicated less standing and writing and more sitting and thinking about math. That is, the PSTs depicted themselves more as students. While the pictures of mathematicians were filled with numerical expressions and numbers, the 26 pictures of PSTs doing math did not contain numbers or symbols (61.9%); of these, 8 images contained geometric ideas (19.0%). In fact, 8 images (19%) did not contain any discernible math. These images generally depicted the PST emoting. Only 12 images (28.6%) contained an equivalence concept. Only 2 images (4.8%) showed the Pythagorean theorem equation. The PSTs drew themselves in a seated position in almost half of the images (20 images, 47.6%) and included desks, tables, and chairs in 18 images (42.9%). Only 16 images (38.1%), depicted the PSTs standing. Compared to the mathematicians, the PSTs were more typically drawn seated, like a student would be in a traditional setting. A calculator was present in 5 drawings (11.9%), while a book appeared in 6 images (14.3%). Furthermore, no image contained more than one individual. Considering the affective factors, 16 images (38.1%) were overall positive, 7 images (16.7%) were neutral, 18 images (42.9%) were overall negative, and 1 image (2.4%) contained both positive and negative elements in equal amounts.

## Students

The drawings of the students were similar in some aspects to the PST images, but key differences existed. In terms of the mathematics, many more images contained basic mathematics involving whole number operations, 22 images of students (52.4%) compared to 10 images of PSTs (23.8%). Additionally, 19 images (45.2%) contained notions of equivalence.


*Figure 1. The most negative and the most positive images.*

Furthermore, 18 images (42.9%) displayed no mathematical symbols. The lack of symbols could in part be explained by 10 drawings (23.8%) depicting children using manipulatives. Students were drawn as sitting (21, 50%) or standing (20, 48.8%) in similar rates compared to the PSTs, with 23 drawings (54.8%) containing desks, tables, or chairs. Significantly, 13 images of students doing math contained two or more people. The prompt did request for "one of your students," so having more than one individual drawn does have significance since only 1 drawing of the mathematicians and PSTs doing math combined depicted more than one individual. Finally, 28 images (66.7%) of students doing math were positive in nature, 11 images (26.2%) were neutral toward math, 2 images (4.8%) were negative, and 1 image (2.4%) contained both positive and negative elements.

## Comparison of Affect

Each image was assigned a score from 1 to 7 using the Bachman, et al. affective scale. The mean score for the mathematicians doing math, the PSTs doing math, and the students doing math were 4.52, 3.79, and 4.86, respectively. That is, the images of mathematicians and students were more positive than negative, while the images of PSTs doing math were more negative. From the Wilcoxson signed rank test, the *p*-values for mathematicians versus PSTs, mathematicians versus students, and PSTs versus students were .0060, .1031, and .0004, respectively. Thus, the drawings of PSTs were significantly more negative than the drawings of mathematicians and students. However, the drawings of mathematicians and students were not significantly different when comparing affect.

While the PSTs appeared to view math more negatively, when depicting their future students doing math, their students appeared to be enjoying the mathematics. As one PST wrote in her analysis, “I believe that my student doing math does look more like the mathematician doing math because the student [is] happy … I believe this is the case because even though I as a teacher do not like math I want my students to enjoy it.” Interestingly, Figure 1 depicts the most negative image of a PST and the most positive image of a student, drawn by the same participant. While some PSTs admitted not liking math in their comments, they also stated that they wanted their students to enjoy math. However, unless the PSTs are able to change their attitude, a tension could exist between their hopes for their students and reality as research seems to indicate that teachers transfer beliefs about math to students (e.g., Mewborn & Cross, 2007).

That the students mirrored the mathematicians in an enjoyment of math seemed to support the idea that students can be mathematicians in their own way, at their own level of beginning mathematics. After all, according to PSTs, mathematicians enjoy doing math (Wescoatt, 2016). A PST even commented, “My picture … shows the student doing math on her level just like the pictures of the mathematician doing math.” However, PSTs appeared to not have this view of themselves as potential mathematicians, seemingly resigned to their fate of never liking math. Chick and Stacey’s view of teachers of math as applied mathematicians would seemingly never apply to PSTs with this negative view. This inability to identify with mathematicians was also evidenced by the PSTs drawing themselves as students; they did not yet identify themselves as a teacher.

Mathematics within the pictures adhered to the stereotype of being a solo endeavor, as long as the actor was an adult. Only one image of a mathematician or a PST included more than one person. In contrast, 13 images of students did. Of these images, 8 contained solely children, with 4 of these showing the children actually interacting with each other. Of the 5 images containing children and adults, 4 images were of a teacher in the classroom asking a student a question. Thus, relatively few images actually depicted people engaging in mathematics together. That the same equations appeared in several images ($E = mc^2$ and $a^2 + b^2 = c^2$), the setting was largely a classroom, the mathematician was a teacher, and mathematics was being done mostly as a solitary endeavor seemed to indicate that due to an immature understanding of what mathematics is, the PSTs resorted to stereotypes of mathematics as they had experienced it up to that point in their school careers. That is, they were drawing what they knew.

## Implications

In their study of images, Picker and Berry (2000) saw a similar reliance on stereotypes by the students in their study. They hypothesized that many of these stereotypes were acquired from teachers and went unchallenged by an alternate view of mathematics. In order for PSTs to not pass on stereotyped views to their future students, their views need to be challenged. Such an intervention could be accomplished by having PSTs share in the analysis of their drawings as in this study. To challenge beliefs of high school students, Latterell and Wilson (2012) brought graduate students in mathematics to the class to discuss what they did with mathematics. Perhaps a similar tact could be taken with PSTs. Based on the pictures and comments, PSTs have high goals for their students, as reflected by the comment, "I want all of my students to learn to love math, and hopefully some of them will grow up to be mathematicians." Whether or not the PSTs could actually help their future students realize these goals may depend on interventions to help them better appreciate math, view it in a more positive way, and see themselves as mathematicians in their own right.

## References


Bachman, R., Berezay, K., & Tripp, L. (2016). Draw yourself doing mathematics: Assessing a mathematics and dance class. In K. V. Adolphson & T. M. Olson (Eds.), *Proceedings of the 43rd Annual Meeting of the Research Council on Mathematics Learning*, (pp. 50-57), Orlando, FL.

Burton, L. (2002). Recignising commonalities and reconciling differences in mathematics education. *Educational Studies in Mathematics, 50*(2), 157-175.

Burton, M. (2012). What is math? Exploring the perception of elementary pre-service teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal, 5*. Retrieved from ERIC database. (EJ970350)

Chick. H., & Stacey, K. (2013). Teachers of mathematics as problem-solving applied mathematicians. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 121-136.

Devlin, K. (2003). *Mathematics: The science of patterns*. New York: Henry Holt and Company.

Farland-Smith, D. (2012). Development and field test of the modified draw-a-scientist test and the draw-a-scientist rubric. *School Science and Mathematics, 112*(2), 109-116.

Latterell, C. M., & Wilson, J. L. (2012). Students' perceptions of what mathematicians do. *The Mathematics Educator, 13*(2), 73-84.

Mewborn, D. S., & Cross, D. I. (2007). Mathematics teachers' beliefs about mathematics and links to students' learning. In W. G. Martin, M. E. Strutchens, & P. C. Elliott (Eds.), *The learning of mathematics: Sixty-ninth yearbook* (pp. 259-269). Reston, VA: National Council of Teachers of Mathematics.

Mitchell, C., Theron, L., Stuart, J., Smith, A., & Campbell, Z. (2011). Drawings as research method. In L. C. Theron, C. Mitchell, J. Stuart, & A. Smith (Eds.) *Picturing research: Drawing as visual methodology* (pp. 19-36), Rotterdam: Sense Publishers.

Picker, S. H., & Berry, J. S. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics*, 43(1), 65-94.

Rule, A. C., & Harrell, M. H. (2006). Symbolic drawings reveal changes in preservice teacher mathematics attitudes after a mathematics methods course. *School Science and Mathematics, 106*(6), 241-258.

Schoenfeld, A. H. (1994). Reflections on doing and teaching mathematics. In A. H. Schoenfeld (Ed.), *Mathematical thinking and problem solving* (pp. 53-70). Hillsdale, NJ: Erlbaum.

Wescoatt, B. (2016). Exploring pre-service teachers' mental models of "doing math". In T. Fukawa-Connelly, N. Infante, M. Wawro, & S. Brown (Eds.), *Proceedings of the 19th Annual Conference on Research in Undergraduate Mathematics Education* (pp. 456-470), Pittsburgh, PA.

# EXPLORING THE EFFECTS OF FOUR YEARS OF A MATH CIRCLE ON MIDDLE SCHOOL AND HIGH SCHOOL STUDENTS' MATHEMATICAL TASK VALUE

Brandy Wiegers, Central Washington University
Yvonne Lai, University of Nebraska-Lincoln
Diana White, University of Colorado-Denver

*Math Circles are a form of informal education where mathematics professionals share their passion for mathematics with K-12 students, combining significant content with an atmosphere that encourages a sense of discovery and excitement about mathematics through problem solving and interactive exploration. Ideal problems offer a variety of entry points and can be approached with minimal mathematical background, but lead to deep mathematical concepts and can be connected to advanced mathematics. We present details on one local model aimed at underserved urban youth, sharing impacts from four years of quantitative data on their mathematical value and expectancy.*

## Introduction and Objectives of the Study

One national initiative aimed at educating children in mathematical problem solving is that of *Math Circles* – informal learning environments, often facilitated after school, whose primary goal is to engage students in sustained and participatory ways. This initiative has grown dramatically; in 2006, there were 30 identified Math Circle sites in the US, and as of 2016, there are 180 sites. This proliferation suggests that Math Circles meet a national need, and furthermore reaffirms a growing public recognition of the importance of self-efficacy, problem solving, and perseverance in mathematical success (e.g., Noguchi, 2015, March; Bajaj, 2013, December).

We examined the effects of one Math Circle on its participants, who were urban middle and high school students, across four years of the program. The main objective of this study was to address: *What is the impact of the Math Circle program on students' mathematical task value and expectancy*? Our study was motivated by findings that mathematical task value and expectancy have been shown to have broad effects on students' mathematical success. Our study leverages Eccles and colleagues' psychological model of expectancy-value and activity choice (Eccles et al., 1983). Using this model as a theoretical framework, we developed an instrument to assess impact on participants' value and expectancy. In this paper, we review Eccles' and colleagues model, describe the development of the study, and conclude with findings from the study and their implications.

## Theoretical Framework

### Expectancy-Value Theory

*Expectancy* is conceptualized as a person's perceived probability of success at that task. *Achievement task value* is a person's desire to engage in that task. Eccles and colleagues constructed and empirically validated a psychological model of how achievement task value and expectancy impact achievement (Eccles et al., 1983). In short, they found that an individual's achievement at an activity — particularly in terms of performance, persistence, and choice — can be explained by the extent to which an individual values that activity and believes they will succeed at that activity. Expectancy-value theory is also consistent with course-enrollment trends in elementary and college levels (Eccles et al., 1983; Ellis, Fosdick, and Rasmussen, 2016).

These results underscore the important role of expectancy and achievement task value in supporting mathematical success. Our study will examine effects of participation on these values.

### Components of expectancy and achievement task value

To operationalize the constructs of expectancy and achievement task value, we used the decomposition proposed by Eccles et al. (1983), where expectancy has two components, and value has four components. The two components of expectancy are: current expectancy, and future expectancy. *Current expectancy* is how much a person expects at the moment that they will succeed at an activity. *Future expectancy* is expectation of success in the future at that activity. However, because our final instrument only contained one question for each, we treated *expectancy* as one component in the final analysis. Value has four components: attainment value, interest or intrinsic value, utility value, and cost. *Attainment* value is the importance of success to the individual; it involves an individual's identity because it is tied to ways of confirming or expressing aspects important to self. *Interest* value is enjoyment gained; enjoyment is linked to persistence. Interest value is measured for specific tasks or activities, such as "math assignments". *Utility* value is usefulness to future plans such as career plans or course requirements; it is extrinsic compared to intrinsic or interest value. *Cost* is sacrifice for a given task or activity, including emotional cost, other activities one might give up for the sake of the task, and expected effort.

# Methodology

## Site of Study

We examined six different San Francisco Math Circle sites over four years. As stated to instructors and students, these sites all aimed for students to gain mathematical confidence and lifelong engagement in mathematical exploration; to provide a safe mathematical community in which students gain an understanding of mathematics as a discipline of inquiry, increase their mathematical and problem solving competency, increase their mathematical persistence, and increase their likelihood to choose to explore harder mathematical tasks. Site locations were local schools; sites sessions occurred once per week during the academic year. Students met within grade-band of middle or high school. Mathematicians and mathematics graduate students led sessions by facilitating groups of four to six students in solving problems, generally spending six weeks on a chosen mathematical theme. Facilitators designed problems so as to be "low-threshold and high-ceiling"; that is, they offered a variety of entry points requiring little to no mathematical background, while also having the potential to lead to ideas from fields of advanced mathematics (e.g., abstract algebra, topology), and at times, to mathematics research problems in those fields.

## Instrument development

The first two authors designed a survey examining achievement task value and expectancy based on the instruments used by Wigfield and Eccles (2000) and Karabenick and Maehr (2003), resulting in more than sixty questions in the initial survey tool, a total of ten printed pages.

To validate the survey tool, in Spring 2010, a sample survey was given to two different groups of student participants and their teachers. The resulting comments were analyzed. A majority of the feedback from teachers focused on the need for questions that could be answered by students without needing teacher support for vocabulary or context. Populations of participating schools included many English language learners. As such, questions that required no additional explanation were selected over those that required clarification. After removing these questions, we reviewed questions to ensure they consistently measured similar values in one component (such as attainment) for individual students. Inconsistent questions were also removed from the final survey tool, resulting in the final survey tool consisting of thirty-one questions that could be printed as two-columns on a single double-sided piece of paper. Questions were a combination of binary response (scaled as a 0 or 1), Likert scale (scaled between 1 and 5), and short-answer. We focus here on nine questions measuring achievement task values and expectancy.

### Classifying instrument responses and sample

This study involved administering pre-post surveys to a total of 2480 students from grades 6-12 over the course of the four academic years from 2010 to 2014. The surveys were collected during program sessions, as well as from the classrooms of teachers at local school sites who were participating in a grant program aimed at placing mathematics graduate students into K-12 classrooms. These settings allowed data to be collected from two student populations: students who had previously participated in the program and a comparison group of students from the same schools who had not participated previously in the program. We recognize that the comparison group of students may not be a typical comparison group, as their classrooms did have the additional support of a mathematics graduate student. Nonetheless, we use this comparison group to provide a starting point to analyze the impact of the program; our logic was that if we did find differences, then the differences could be more attributed to Math Circle-specific design rather than simply the presence of a professional in mathematics. Teachers were instructed to not give the survey twice to any students who both were in their classroom and had attended Math Circle.

Completed surveys were separated into three groups using program identification questions: *Math Circle participants (participants)*, *non-Math Circle participants (non-participants)*, and *unidentifiable*. For the post-survey, participants were identified from those who indicated regular attendance in Math Circle in the last year. For the pre-survey, returning participants were identified from those who both took the survey at a Math Circle site *and* identified as having regularly attended the previous year's Math Circle, meaning more than half the sessions. Unidentifiable surveys, from students who could not be classified as either participants or non-participants, were removed, as were incomplete surveys. Matched pair-analysis of pre- and post-surveys was not completed due to a flaw in matching code that was discovered later in the data-analysis.

After separation, our sample consisted of 737 from non-participants and 122 pre-survey responses from returning participants, for a total of 859; and 336 from non-participants with 90 post-survey responses from participants, for a total of 426.

## Results and Discussion

Data analysis was completed with a two-sample *t*-test via Minitab. Pre- and post-survey analysis indicated nearly all the values had significant differences for participants, with the exception of attainment. Similar patterns were seen when comparing program participant with non-program participant surveys, where all but one survey item had a statistically significant difference between the two groups ($p < 0.008$). To examine this further, we studied the pre- and post-survey results of the non-participants, comparing the results with the post-survey and returning pre-surveys results of the participants. These results are reported in Table 1 and discussed in context for each component.

In looking at the pre- and post-surveys, similar patterns as with the entire group were seen in the results of non-participants, who showed a statistically significant change over the course of the academic year in all task values but attainment. One limitation of these results comes from the definition of non-participants. In our classification strategy, students who took the survey at a Math Circle site but did *not* indicate regular attendance in the past year were counted as non-participants. Consequently, non-participants included for the pre-survey included first-time Math Circle students, along with students who not attending Math Circle that year. At this time we believe that the results of the returning participants, which are statistically significant different when compared to non-participants, indicate that students who returned from previous program participation maintained high values in their pre-survey attitudinal assessment, which may suggest potential long-term program impact.

Table 1

*Pre- and post-survey achievement task value averages for participants and non-participants. Difference compares survey to pre-non-participant surveys.*

| Task Value | | Non-Participants | | | Participants | | Returned Participants | |
|---|---|---|---|---|---|---|---|---|
| | | Pre | Post | Difference | Post | Difference | Pre | Difference |
| Attainment | Item 1 | 0.82 | 0.77 | -0.05 | 0.90 | 0.08* | 0.89 | 0.07* |
| | Item 2 | 0.94 | 0.94 | 0.00 | 0.94 | 0.01 | 0.93 | -0.01 |
| | Item 3 | 3.62 | 3.65 | 0.03 | 4.00 | 0.38*** | 3.89 | 0.27** |
| Interest | Item 1 | 3.20 | 3.40 | 0.21** | 4.09 | 0.89*** | 3.96 | 0.75*** |
| | Item 2 | 0.53 | 0.64 | 0.12*** | 0.74 | 0.22*** | 0.81 | 0.28*** |
| Utility | Item 1 | 0.90 | 0.86 | -0.05* | 0.97 | 0.06** | 0.93 | 0.03 |
| | Item 2 | 0.84 | 0.84 | 0.00 | 0.94 | 0.11*** | 0.87 | 0.03 |
| Cost | Item 1 | 0.33 | 0.45 | 0.12*** | 0.57 | 0.24*** | 0.54 | 0.22*** |
| | Item 2[a] | 0.51 | 0.43 | -0.08* | 0.27 | -0.24*** | 0.35 | -0.16*** |

[a] Cost Item 2 decreasing is a positive result. See narrative for details.
*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

### Attainment

For three of the four years, Math Circle students were the only group that improved their value related to attainment over the course of the academic year. In looking more closely at the measured items, Item 1 and Item 3 provided the most insight. Item 1 showed a statistically significant difference between the two groups ($p < .001$). Non-participants do not have significant difference in the pre- vs post-survey ($p = 0.090$) while the participant group maintained high attainment values over the year. Note that the average attainment value for the non-participants went from 0.82 to 0.77 while participants increase from 0.88 to 0.90 over the course of the academic year. In Item 3 the two groups did not show significant difference in their pre- and post-survey responses but there is a significant difference between the participants and non-participants ($p < 0.001$) with the participants pre-survey mean of 3.89 and non participant pre-survey of 3.62. This is maintained in the post-survey with participant mean of 4.00 and nonparticipant mean of 3.65.

Interest

Results indicate students in the Math Circle have higher interest in mathematics. There is a significant difference between participants and non-participants for both items ($p < 0.001$), combined with a significant difference between pre- and post-surveys on both items (both $p < 0.001$). The non-participants increased significantly on both items ($\underline{p} < 0.01$ and $p < 0.001$) and while the participants did have significant change on the pre- versus post-surveys, the program participant average is much larger for both. In looking at Item 1, the pre-survey average for non-participants is 3.20 and ends at 3.41 in the post-survey. This compares to the pre-survey of Math Circle participants, which begins at 3.95 and ends at 4.09. Similar patterns occur for Item 2 although Item 2 sees a decrease in participant values. This item measured fear associated with doing mathematical tasks, which should measure as smaller for someone with high mathematical task value. In further discussion with the Math Circle students about these responses it was discovered that these students were frustrated by their in-class experience when compared to their Circle experience, which is addresses in Interest Item 2..

Utility

Math Circle participants rank higher than non-participants in the task value of utility. They increased their value related to problem-solving skills, with the significant difference on both items between participants and non-participants ($p = 0.001$ and $p = 0.0083$, respectively). Students who returned to the Circle in the fall maintained a high task value in the pre-survey, indicating a maintained value of the utility of mathematics. In the last year of the study, this value ranked highest in values for participants, thereby supporting that the program is having a lasting impact on the mathematical attitudes of the students involved.

Cost

Students in the Math Circle demonstrated increases for the cost task value, with a significant difference between participants and non-participants (*p* < 0.001 on both items) and between pre and post-surveys (*p* < 0.001 and *p* = 0.0025, respectively on the two items). Specifically, program students had a reduced negative impact from their participation in math over the course of the year (increasing on Item 1 from 0.54 to 0.57 and decreasing on item 2, as expected, from 0.35 to 0.27). This compares to the other non-program students who did not have the same improvement in regards to cost value (increasing from 0.33 to 0.45 on Item 1 and decreasing from 0.51 to 0.43 on Item 2). Cost Item 2 differs from the other survey items because cost value decreasing over the year aligns with desired program outcomes. There is a significant difference between the participants and non-participants (*p* < 0.001), with a mean value of .31 of for participants and .49 of non-participants. Looking more carefully, the initial mean of the participants was 0.35, dropping to 0.26, while the non-participants start at 0.51 and end at 0.43. For the non-participants this is a significant change (*p* = 0.02). Note that the mean of the pre-survey for the participants is higher than the post-survey for non-participants, connecting to the idea that participants return to the program with a lower level of cost value than non-participants have over the whole year.

### Expectancy

Two survey items were used to study this aspect of Math Circle program impact on students, and both items showed a statistically significant difference between participants and non-participants (*p* < 0.001 for both). There was also a significant difference between the pre- and post-survey results for both items (*p* < 0.001 and *p* = 0.051, respectively). As previously, we then divided the analysis into pre- and post-survey results for participants and non-participants, as seen in Table 2.

Table 2

*Pre- and post-survey expectancy averages for participants/ non-participants. Difference compares survey to pre-non-participant surveys.*

| Expectancy | Non-Participants | | | Participants | | Returned Participants | |
|---|---|---|---|---|---|---|---|
| | Pre | Post | Difference | Post | Difference | Pre | Difference |
| Item 1 | 3.21 | 3.39 | 0.19* | 3.77 | 0.56*** | 3.54 | 0.34** |
| Item 2 | 3.91 | 3.77 | -0.14* | 4.11 | 0.20* | 4.25 | 0.33*** |

*$p < 0.05$, **$p < 0.01$, ***$p < 0.001$

We observe that Math Circle participants end and return with a higher expectancy value. Looking specifically at Item 1, non-participants start at 3.21 and end at 3.39. This compares to participants who end at 3.77 and return at 3.54. For Item 2, we see a decrease in both groups when looking at the post-survey.

## Implications

We compared the pre- and post- achievement task value and expectancy of Math Circle students with students who had not participated or who had not yet participated in a Math Circle. We found differences across all components of achievement task value and expectancy. In particular, we found that Math Circle participants' post-attainment and post-utility is higher than non-participants', and that, moreover, non-participants' post-attainment and post-utility is actually lower than their pre-attainment and pre-utility, whereas participant's post-attainment and post-utility is higher than their pre-attainment and pre-utility. To put this result in terms of the "average" participant and non-participant, our findings suggest that if Kim is a long-term regular Math Circle participant, then she is more likely to increase how much she wants to do well at math for both intrinsic (attainment) and extrinsic (utility) reasons, and that this effect may accumulate over time. In contrast, non-participants are likely to slightly decrease over time in wanting to do well in math, and this effect may also accumulate over time.

One limitation of this study is due the classification of first-time Math Circle student as non-participants and the lack of matched survey responses. We were thus unable to analyze impacts of Math Circle on individuals beginning in their first year of participation. A second limitation of this study is its scope. Although the Math Circle initiative is certainly consistent with the aim of increase achievement task value and expectancy of its students, the method in which it accomplishes this is indirect. In this paper, we focused on closed-ended survey results on task value and expectancy, and did not address how a Math Circle shapes students' conception of mathematics and mathematical professionals. Initial analyses of an open-ended survey response indicate that participants enjoy the program and are provided with an opportunity to think broadly and expand their problem solving skills. They value the social aspect and appreciate the chance to interact with mathematicians and other peers who are passionate about mathematics. In addition, these participants diversified their descriptions of mathematicians to include tattoos, women, and more people with an outward appearance similar to the students.

This study indicates that participating in Math Circle may have impacts that accumulate over time, in traits that may well impact students' career decisions. Directions suggested by this study include how Math Circles impact students at the individual level and by years of participation, and how these effects are related to students' conception of mathematics.

## References

Eccles, J. S., with Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J. L., & Midgley, C. (1983). Expectancies, values, and academic behaviors. In J. T. Spence (Ed.), *Achievement and achievement motives: Psychological and sociological approaches*. San Francisco, CA: W. H. Freeman & Co.

Ellis, J., Fosdick, B. K., Rasmussen, C. (2016). Women 1.5 Times More Likely to Leave STEM Pipeline after Calculus Compared to Men: Lack of Mathematical Confidence a Potential Culprit. *PLoS ONE, 11*(7): e0157447. doi:10.1371/journal.pone.0157447

Karabenick, S. A., & Maehr, M. L. (Eds.*). MSP-motivation assessment program: Tools for the evaluation of motivation-related outcomes of math and science instruction, Final Report to the National Science Foundation, NSF EHR 0335369.* Retrieved from http://hub.mspnet.org/index.cfm/15222

Wigfield, A., & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology, 25*, 68–81.

Noguchi, S. (2015, March 17) Hate math? It's not about you, and new teaching methods may help. The Mercury News. Retrieved from http://www.mercurynews.com/2015/03/17/hate-math-its-not-about-you-and-new-teaching-methods-may-help/

Bajaj, V. (2013, December 12). Q. & A. with Carol Dweck. New York Times. Retrieved from http://www.nytimes.com/2013/12/12/opinion/q-a-with-carol-s-dweck.html

## Acknowledgements

This research was supported by the National Science Foundation through a GK12 Program (grant DGE-0841164), the Bechtel Foundation, and Moody's Foundation.