

Problems and Solutions

Albert Natian, Section Editor

This section of the SSMA Journal offers readers an opportunity to exchange interesting mathematical problems and solutions. Please send them to Prof. Albert Natian, Department of Mathematics, Los Angeles Valley College, 5800 Fulton Avenue, Valley Glen, CA, 91401, USA. It's highly preferable that you send your contributions via email.

To propose problems, email them to: **problems4ssma@gmail.com**

To propose solutions, email them to: **solutions4ssma@gmail.com**

Please follow the instructions for submission of problems and solutions provided at the end of this document. Your adherence to all submission requirements is of the greatest help in running this Section successfully. **Thank you!**

Solutions to previously published problems can be seen at <www.ssma.org/publications>.

Solutions to the problems published in this issue should be submitted before May 1, 2023.

• **5715** Proposed by *Kenneth Korbin, New York, NY.*

Find the dimensions of a triangle with integer length sides in which the area divided by the perimeter is equal to the sine of 60 degrees.

• **5716** Proposed by *Daniel Sitaru, National Economic College "Theodor Costescu" Drobeta Turnu - Severin, Romania.*

Prove:

$$\text{If } x, y \in \mathbb{R}, \text{ then } |\cos x \cos y \sin(x + y)| \leq \frac{3\sqrt{3}}{8}.$$

• **5717** Proposed by *Rafael Jakimczuk, Departamento de Ciencias Básicas, División Matemática, Universidad Nacional de Luján, Buenos Aires, Argentina.*

Let h_1 and h_2 be nonnegative integers. Prove the following limit:

$$\lim_{n \rightarrow \infty} \left(\frac{\prod_{k=1}^n \binom{n+h_1}{k}}{\prod_{k=1}^n \binom{n+h_2}{k}} \right)^{\frac{1}{n}} = e^{h_1-h_2}.$$

• **5718** Proposed by *José Luis Díaz-Barrero, BarcelonaTech, Barcelona, Spain.*

Find all real solutions of the system of equations

$$\left\{ \begin{array}{l} x^4 + 25y^2 + 12 = 8y^3 + 28z + 4x\sqrt{x-1} \\ y^4 + 25z^2 + 12 = 8z^3 + 28x + 4y\sqrt{y-1} \\ z^4 + 25x^2 + 12 = 8x^3 + 28y + 4z\sqrt{z-1} \end{array} \right\}.$$

- **5719** Proposed by Toyesh Prakash Sharma, Agra College, Agra, India.

Prove the compound inequality

$$\sqrt{2} < \int_0^1 x^x dx + \left(\int_0^1 x^{-x} dx \right)^{-1} < 2.$$

- **5720** Proposed by Raluca Maria Caraion, Călărași, Romania and Florică Anastase, Lehliu-Gară, Romania.

Let $(a_n)_{n \geq 1}$ be sequence of real numbers such that $\forall n \geq 1 : a_n \leq n$ and

$$\forall n \geq 2 : \sum_{k=1}^n \cos \left(\frac{\pi a_k}{n} \right) = 0.$$

Find

$$\Omega = \lim_{n \rightarrow \infty} \left(a_n \cdot \sum_{k=0}^{2n} (-1)^k \frac{\binom{4n}{2n-k}}{\binom{2n}{k}} \right).$$

Please note: Solutions to problems 5697 through 5702 will be published in a future issue of the Problems/Solutions Section.

Editor's Statement: It goes without saying that the problem proposers, as well as the solution proposers, are the *élan vital* of the Problems/Solutions Section of SSMJ. As the editor of this Section of the Journal, I consider myself fortunate to be in a position to receive, compile and organize a wealth of proposed ingenious problems and solutions intended for online publication. My unwavering gratitude goes to all the amazingly creative contributors. We come together from across continents because we find intellectual value, joy and satisfaction in mathematical problems, both in their creation as well as their solution. So that our collective efforts serve us well, I kindly ask all contributors to adhere to the following guidelines. As you peruse below, you may construe that the guidelines amount to a lot of work. But, as the samples show, there's not much to do. Your cooperation is much appreciated!

Keep in mind that the examples given below are your best guide!

Formats, Styles and Recommendations

When submitting proposed problem(s) or solution(s), please send both **LaTeX** document and **pdf** document of your proposed problem(s) or solution(s). There are ways (discoverable from the internet) to convert from Word to LaTeX.

Regarding Proposed Solutions:

Below is the FILENAME format for all the documents of your proposed solution(s).

#ProblemNumber_FirstName_LastName_Solution_SSMJ

- FirstName stands for YOUR first name.
- LastName stands for YOUR last name.

Examples:

#1234_Max_Planck_Solution_SSMJ

#9876_Charles_Darwin_Solution_SSMJ

Please note that every problem number is *preceded* by the sign # .

All you have to do is copy the FILENAME format (or an example below it), paste it and then modify portions of it to your specs.

Please adopt the following structure, in the order shown, for the presentation of your solution:

1. On top of the first page of your solution, begin with the phrase:

“Proposed Solution to #**** SSMJ”

where the string of four astrisks represents the problem number.

2. On the second line, write

“Solution proposed by [your First Name, your Last Name]”,

followed by your affiliation, city, country, all on the same linear string of words. Please see the example below. Make sure you do the same for your collaborator(s).

3. On a new line, state the problem proposer’s name, affiliation, city and country, just as it appears published in the Problems/Solutions section.

4. On a new line below the above, write in bold type: “**Statement of the Problem**”.

5. Below the latter, state the problem. Please make sure the statement of your problem (unlike the preceding item) is not in bold type.

6. Below the statement of the problem, write in bold type: “**Solution of the Problem**”.

7. Below the latter, show the entire solution of the problem.

Here is a sample for the above-stated format for proposed solutions:

Proposed solution to #1234 SSMJ

Solution proposed by Emmy Noether, University of Göttingen, Lower Saxony, Germany.

Problem proposed by Isaac Newton, Trinity College, Cambridge, England.

Statement of the problem:

Compute $\sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$.

Solution of the problem:

Regarding Proposed Problems:

For all your proposed problems, please adopt for all documents the following FILENAME format:

FirstName_LastName_ProposedProblem_SSMJ_YourGivenNumber_ProblemTitle

If you do not have a ProblemTitle, then leave that component as it already is (i.e., ProblemTitle).

The component YourGivenNumber is any UNIQUE 3-digit (or longer) number you like to give to your problem.

Examples:

Max_Planck_ProposedProblem_SSMJ_314_HarmonicPatterns

Charles_Darwin_ProposedProblem_SSMJ_358_ProblemTitle

Please adopt the following structure, in the order shown, for the presentation of your proposal:

1. On the top of first page of your proposal, begin with the phrase:

“Problem proposed to SSMJ”

2. On the second line, write

“Problem proposed by [your First Name, your Last Name]”,

followed by your affiliation, city, country all on the same linear string of words. Please see the example below. Make sure you do the same for your collaborator(s) if any.

3. On a new line state the title of the problem, if any.

4. On a new line below the above, write in bold type: “**Statement of the Problem**”.

5. Below the latter, state the problem. Please make sure the statement of your problem (unlike the preceding item) is not in bold type.

6. Below the statement of the problem, write in bold type: “**Solution of the Problem**”.

7. Below the latter, show the entire solution of your problem.

Here is a sample for the above-stated format for proposed problems:

Problem proposed to SSMJ

Problem proposed by Isaac Newton, Trinity College, Cambridge, England.

Principia Mathematica (←— You may choose to not include a title.)

Statement of the problem:

Compute $\sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$.

Solution of the problem:

♣ ♣ ♣ **Thank You!** ♣ ♣ ♣